OLD DOMINION UNIVERSITY RESEARCH FOUNDATION

DEPARTMENT OF MECHANICAL ENGINEERING AND MECHANICS
SCHOOL OF ENGINEERING
OLD DOMINION UNIVERSITY
NORFOLK, VIRGINIA 23508

THREE-DIMENSIONAL ELASTIC-PLASTIC FINITE-ELEMENT ANALYSIS OF FATIGUE CRACK PROPAGATION

By

R. G. Chermahini, Research Associate

and

G. L. Goglia, Principal Investigator

Final Report
For the period June 1, 1985 to November 1, 1985

Prepared for
National Aeronautics and Space Administration
Langley Research Center
Hampton, VA 23665

Under
Research Grant NAG-1-529
Dr. James C. Newman, Jr., Technical Monitor
MD-Fatigue & Fracture Branch

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THREE-DIMENSIONAL ELASTIC-PLASTIC FINITE-ELEMENT ANALYSIS OF FATIGUE CRACK PROPAGATION

BY

R. G. Chermahini[1] and G. L. Goglia[2]

INTRODUCTION

Fatigue cracks have been a major problem in designing structures subjected to cyclic loading. Cracks frequently occur in structures such as aircraft and spacecraft. The inspection intervals of many aircraft structures are based on crack-propagation lives. Therefore, improved prediction of propagation lives under flight-load conditions (variable-amplitude loading) are needed to provide more realistic design criteria for these structures.

The main thrust of this study was to develop a three-dimensional, nonlinear, elastic-plastic, finite element program capable of extending a crack and changing boundary conditions for the model under consideration. The finite-element model is composed of 8-noded (linear-strain) isoparametric elements. In the analysis, the material is assumed to be elastic-perfectly plastic. The cycle stress-strain curve for the material is shown in Fig. 1. Zienkiewicz's "initial-stress" method, von Mises's yield criterion, and Drucker's normality condition under small-strain assumptions are used to account for plasticity. The three-dimensional analysis is capable of extending the crack and changing boundary conditions under cyclic loading. Initially, the crack is assumed to grow as a straight-through crack.

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Using a three-dimensional nonlinear computer program on a cyber-nos system was impossible due to its limited storage capacity. To avoid this problem, the next alternative was to utilize a VPS-32 machine with unlimited storage capacity. Using the scalar version of the program on the VPS-32 was costly due to the plasticity part of the program. Therefore, in order to reduce the cost of the computations, the three-dimensional computer program was vectorized.

The finite-element formulation of the program using an 8-noded linear isoparametric cubic element is listed in Appendix A. The description of the nonlinear program is attached in Appendix B. A list of the program is shown in Appendix C.

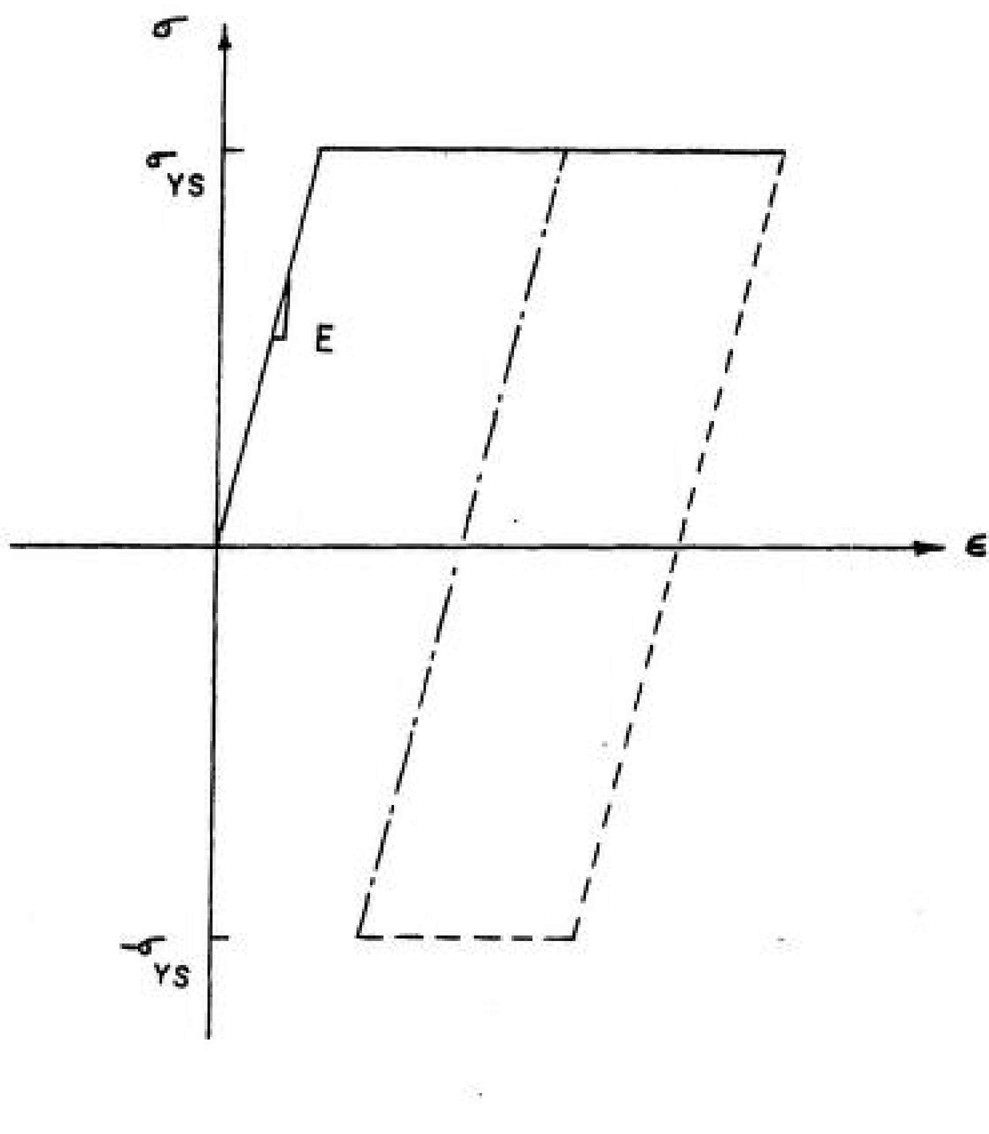


Figure 1. CYCLIC STRESS-STRAIN CURVE FOR AN ELASTIC-PERFECTLY PLASTIC MATERIAL

APPENDIX A

FINITE-ELEMENT FORMULATION

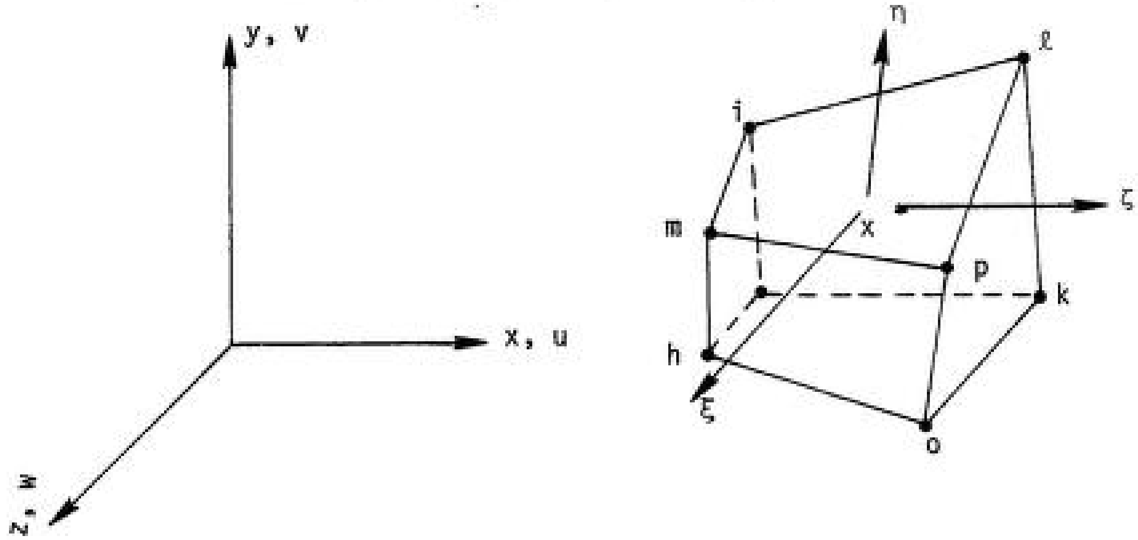


Figure 2. Arbitrary hexahedron.

(a) Elastic Analysis

The basic concept of the finite-element method is that any continuous quantity can be approximated by a discrete model composed of a set of piecewise continuous functions defined over a finite number of subdomains (1).

An isoparametric 8-noded cubic element (Fig. 2.) was utilized in the formulation of the elastic-plastic structure into the nonlinear computer program. The following section describes cubic element.

Displacement Functions. The displacement function (2) for any point in the cubic element is defined as:

$$u(x,y,z) = a_1 + a_2x + a_3y + a_4z + a_5xy + a_6yz + a_7xz + a_8xyz$$
$$v(x,y,z) = b_1 + b_2x + b_3y + b_4z + b_5\ xy + xy + b_6yz + b_7xz + b_8xyz$$
$$w(x,y,z) = c_1 + c_2\ x\ c_3y + c_4z + c_5xy + c_6yz + c_7xz + c_8xyz \qquad A(1)$$

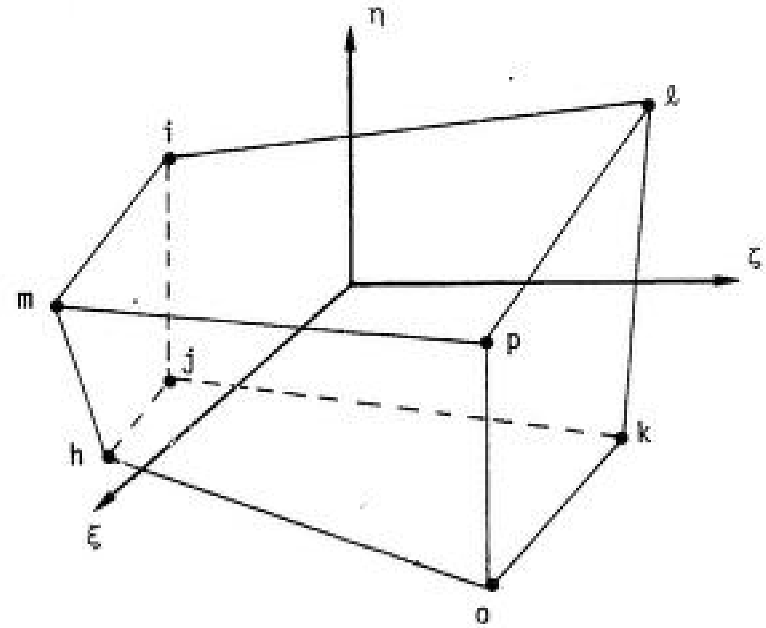
where u, v and w are displacement in the x, y and z directions, respectively. The constant coefficients are determined by imposing the nodal coordinates of each cubic element into equations A (1). The above displacement function can be applied to the cubic element as long as the sides of the cubic element are defined by planes parallel to the coordinate planes. However, for the elements whose sides are skewed, the above displacement function no longer is applicable. Therefore, in order to avoid this restriction, an 8-noded linear isoparametric cubic element is employed (Fig. 2.).

The original cube can be mapped on to a cube of 2x2x3 unit (2) in the ζ, η, ξ space by the transformation

$$x = a_1 + a_2\zeta + a_3\eta + a_4\xi + a_5\zeta\eta + a_6\eta\xi + a_7\zeta\xi + a_8\zeta\eta\xi$$
$$y = b_1 + b_{2\zeta} + b_{3\eta} + b_{4\xi} + b_{5\xi\eta} + b_{6\eta\xi} + b_{7\zeta\xi} + b_{8\zeta\eta\xi} \qquad A(2)$$
$$z = c_1 + c_{2\zeta} + c_{3\eta} + c_{4\xi} + c_{5\xi\eta} + c_{6\eta\xi} + c_{7\zeta\xi} + c_8\zeta\eta\xi$$

The values of the coefficients in equation A(2) depend on the nodal coordinates of each cubic element and are different for different elements. The transformation is defined by polynomials in ζ, η and ξ which is continuous within the element, the continuum confined within an element in x, y and z coordinates is mapped on to a continuum within the 2x2x2 cube in ζ, η and ξ coordinates. It remains to be shown that the transformation is continuous across two adjoined elements, that a common surface between two adjoined elements in the x, y, z space will transform into a common surface of two adjoined cubes in ζ, η, ξ space.

If we assign the following values of the parameters ζ, η, ξ to the faces of distorted elements shown in (Fig. 3.) one yields:



Face	Coordinate value
pokl	$\zeta = 1$
mnji	$\zeta = 1$
impl	$\eta = 1$
jnok	$\eta = -1$
mnop	$\xi = 1$
ijkl	$\xi = 1$

Fig. 3. Linear isoparametric cubic element.

Therefore, the nodal points i, j, k, l and m, n, o, p will have the following coordinates in the ζ, η, ξ:

nodal point	coordinates		
i	$\zeta_i = -1$	$\eta_i = 1$	$\xi_i = -1$
j	$\zeta_j = -1$	$\eta_j = -1$	$\xi_j = -1$
k	$\zeta_k = 1$	$\eta_k = -1$	$\xi_k = -1$
l	$\zeta_l = 1$	$\eta_l = 1$	$\xi_l = -1$
m	$\zeta_m = -1$	$\eta_m = 1$	$\xi_m = 1$
n	$\zeta_n = -1$	$\eta_n = -1$	$\xi_n = 1$
o	$\zeta_o = 1$	$\eta_o = -1$	$\xi_o = 1$
p	$\zeta_p = 1$	$\eta_p = 1$	$\xi_p = 1$

Now the displacements (u, v, w) in the x, y, z directions can be written as:

$$
\begin{aligned}
u &= \alpha_1 + \alpha_2\zeta + \alpha_3\eta + {}_4\xi + \alpha_5\xi\eta + \alpha_6\eta\xi + \alpha_7\zeta\xi + \alpha_8\zeta\eta\xi \\
v &= \beta_1 + \beta_2\xi + \beta_3\eta + \beta_4\xi + \beta_5\zeta\eta + \beta_6\eta\xi + \beta_7\zeta\xi + \beta_8\zeta\eta\xi \qquad A(3) \\
w &= \gamma_1 + \gamma_2\zeta + \gamma_3\eta + \gamma_4\xi + \gamma_5\zeta + \gamma_6\eta\xi + \gamma_7\zeta\xi + \gamma_8\zeta\eta\xi
\end{aligned}
$$

which are continuous (2) within the elements as well as across the surfaces common to any two adjoined elements. Consider the term u in equations A(3), denote by $\{a\}$ and $\{U\}$ the vectors for the α's and u_i nodal displacements of all the nodal points of the element. Inserting the values of u_i, ζ_i, η_i and ξ_i for the various nodal points, we obtain eight equations corresponding to the first equation of A(3) which can be written as

$$\{U\} = [A_1]\,\{a\} \qquad A(4)$$

Let's define $[\alpha_1] = [A_1]$, and thus have $\{a\} = \lfloor\alpha_1\rfloor \{u\}$. Now the displacement functions for the distorted element can be written as:

$$
\begin{aligned}
u &= \lfloor S \rfloor \; [\alpha_1] \{u\} \\
v &= \lfloor S \rfloor \; [\alpha_1] \{v\} \\
w &= \lfloor S \rfloor \; [\alpha_1] \{w\}
\end{aligned}
\qquad A(5)
$$

where $[S]$ is defined as:

$$\lfloor S \rfloor = \lfloor 1 \;\; \zeta \;\; \eta \;\; \xi \;\; \zeta\eta \;\; \eta\xi \;\; \zeta\xi \;\; \zeta\eta\xi \rfloor \qquad A(6)$$

The shape functions for the isoparametric 8-noded element can be determined (1) from the product of [S] and $[\alpha_1]$ matrices.

$$N_i = \frac{1}{8}\,(1+\zeta_i)\;(1+\eta_i)\;(1+\xi_i) \qquad A(7)$$

where ζ_i, η_i, $\xi_i = \pm 1$ and i = 1, 2, ..., 8.

The x, y, and z coordinates at any point in the element, can be expressed in terms of shape functions N_i:

$$
\begin{aligned}
x &= \sum_{i=1}^{8} N_i \, X_i \\
y &= \sum_{i=1}^{8} N_i \, y_i \\
z &= \sum_{i=1}^{8} N_i \, z_i
\end{aligned}
\qquad A(8)
$$

or

$$\begin{Bmatrix} x \\ y \\ z \end{Bmatrix} = \begin{vmatrix} [N] & [0] & [0] \\ [0] & [N] & [0] \\ [0] & [0] & [N] \end{vmatrix} \begin{Bmatrix} \{x_n\} \\ \{y_n\} \\ \{z_n\} \end{Bmatrix}$$

where $\{x_n\}^T = [x_1 \ x_2 \ \ldots \ x_8]$, $\{y_n\}^T = [y_1 \ y_2 \ \ldots \ y_8]$

and $\{z_n\}^T = [z_1 \ z_2 \ \ldots \ z_8]$.

Element Strain: The elastic strain at any point within the element is given by [3]

$$\{\varepsilon\} = \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{zx} \end{Bmatrix} = \begin{Bmatrix} \frac{\partial u}{\partial x} \\ \frac{\partial v}{\partial y} \\ \frac{\partial w}{\partial z} \\ \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \\ \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \\ \frac{\partial w}{\partial x} + \frac{\partial u}{\partial z} \end{Bmatrix} = \Big[[B_1] \ [B_2] \ldots [B_8] \Big] \{u\} = [B] \{u\} \qquad A(9)$$

where the matrix [B] is defined as:

$$[B_i] = \begin{bmatrix} \frac{\partial N_i}{\partial x} & 0 & 0 \\ 0 & \frac{\partial N_i}{\partial y} & 0 \\ 0 & 0 & \frac{\partial N_i}{\partial y} \\ \frac{\partial N_i}{\partial y} & \frac{\partial N_i}{\partial x} & 0 \\ 0 & \frac{\partial N_i}{\partial z} & \frac{\partial N_i}{\partial y} \\ \frac{\partial N_i}{\partial z} & 0 & \frac{\partial N_i}{\partial x} \end{bmatrix} \qquad A(10)$$

The transformation relationship between local and global coordinates is given by:

$$\begin{Bmatrix} \frac{\partial N_i}{\partial x} \\ \frac{\partial N_i}{\partial y} \\ \frac{\partial N_i}{\partial z} \end{Bmatrix} = [J]^{-1} \begin{Bmatrix} \frac{\partial N_i}{\partial \zeta} \\ \frac{\partial N_i}{\partial y} \\ \frac{\partial N_i}{\partial \xi} \end{Bmatrix} \qquad A(11)$$

where [J] is the Jacobian matrix and it is defined as:

$$[J] = \begin{bmatrix} \frac{\partial \{N\}^T}{\partial \xi} \\ \frac{\partial \{N\}^T}{\partial \eta} \\ \frac{\partial \{N\}^T}{\partial \xi} \end{bmatrix} \{x_n\} \ \{y_n\} \ \{z_n\} \qquad A(12)$$

where $\{x_n\}^T = [x_1 \ x_2 \ \dots \ x_8]$.

Element Stress. For linear-elastic and isotropic materials, the element stresses are calculated using Hook's law

$$\{\sigma\} = [D]\{\varepsilon\} + \{\mathring{\sigma}\} \qquad A(13)$$

The strain vector is $\{\varepsilon\} = [B]\{u\}$; therefore, the stresses are

$$\{\sigma\} = [D][B]\{u\} + \{\mathring{\sigma}\} \qquad A(14)$$

where $\{\sigma^{o}\}$ is initial stress which may exist in the element. The material property matrix [D], is defined as:

$$[D] = \frac{E}{(1+\nu)(1-2\nu)} \begin{vmatrix} 1-\nu & \nu & \nu & o & o & o \\ & 1-\nu & \nu & o & o & o \\ & & 1-\nu & o & o & o \\ \text{Symmetrical} & & & \frac{1-2\nu}{2} & o & o \\ & & & & \frac{1-2\nu}{2} & o \\ & & & & & \frac{1-2\nu}{2} \end{vmatrix} \qquad A(15)$$

where E is Young's modulus and ν is poisson's ratio for the material.

Element Equations. The potential energy π_p (u, v, w) which is composed of strain energy u_p (u, v, w) and v_p (u, v, w) the work done by the applied loads during displacement changes is given by [3].

$$\pi_p = \frac{1}{2} \iiint_v \{\varepsilon\}^T [D] \{\varepsilon\} dv - \iiint_v [F^*] \{\bar{\delta}\} dv + \iint_{s_1} [T^*] \{\bar{\delta}\} ds \qquad A(16)$$

where $[F^*] = [x^* y^* z^*]$, $[T^*] = [T_x^* \; T_y^* \; T_z^*]$.

and $[\bar{s}] = [u,v,w]$.

The equilibrium equations for the element are obtained by taking partial derivatives of π_p with respect to u_1, v_1, w_1, etc., and equating to zero,

$$\frac{\partial \pi_p}{\partial \{u\}} = 0 \quad , \qquad A(17)$$

which leads to the 24 element equilibrium equations as

$$\underset{24x24}{[K]} \; \underset{24x1}{\{u\}} = \underset{24x1}{\{q\}} = \underset{24x1}{\{q_1\}} + \underset{24x1}{\{q_2\}} \qquad A(18)$$

where [K] is the element stiffness matrix,

$$[k] = \iiint_v [B]^T [D] [B] dv + [K_s] \qquad A(19)$$

and {Q} is the element nodal load vector,

$$\{Q\} = \{Q_1\} + \{Q_2\} = \iiint_V [N]^T \{F^*\}\, dv + \iint_{S_1} [N]^T \{T^*\}\, ds \qquad A(20)$$

The diagonal matrix $[K_s]$ in Eq. A(19) is the eleastic stiffness of the springs, which are connected to the boundary nodes.

(b) Elastic-plastic analysis

Finite-element techniques applied to linear elastic materials have been solved successfully. However, for an elastic-plastic material, the coefficient in the stiffness matrix varies as a function of material loading. Two computational methods have been used successfully in the solution of elastic-plastic problems. In the first, the change at each step of load increase in plastic strain is calculated and treated as an initial strain for which the elastic stress distribution is adjusted (1). This method fails if ideal plastic is postulated or if the degree of hardening is small. In the second method, the "incremental stress method," the stress-strain relationship for every load increment is adjusted to account for plastic deformations. The work of Pope (4), Swedlow (5), Marcal and King (6), Reyes and Deere (7) and Popov and others (8) falls into this category.

The "incremental elasticity" method has one serious disadvantage. At each step of the computation the stiffness matrix of the structure is updated and iterative schemes of solution are necessary to avoid excessive computational costs. To minimize computational costs, the "initial stress" approach is used (1). In the incremental stress method, the basic elasticity matrix remains unchanged. This technique converges more rapidly than the initial strain method.

Yield Criterion. In any elastic-plastic analysis, it is necessary to introduce a yield criterion to determine the state of stress at which yielding occurs. The von Mises yield criterion or maximum distortion energy theory of failure, which finds considerable experimental support in ductile materials, is used to determine whether the material at any point in the structure has yielded. This criterion assumes that yielding begins when the distortion energy equals the distortion energy at yield in simple tension (1). The von Mises yield criterion for a three dimensional state of stress is given by

$$F = F(\sigma) = \left[\frac{1}{2}(\sigma_x - \sigma_y)^2 + \frac{1}{2}(\sigma_y - \sigma_z)^2 + \frac{1}{2}(\sigma_z - \sigma_x)^2 + 3T_{xy}^2 + 3T_{xz}^2 + 3T^2_{xz}\right]^{\frac{1}{2}} - \bar{\sigma} \qquad A(21)$$

where $\bar{\sigma} = \bar{\sigma}(K)$ is the uniaxial stress at yield. If $F(\sigma) < 0$, the material is in elastic range. If $F(\sigma) \geq 0$, the material has experienced plastic deformation and one of the flow theories of plasticity must be used for determining the components of plastic strains and stresses due to the applied load.

During an infinitesimal increment of stress, changes of strain are assumed to be divisible into elastic and plastic parts (1). Thus, the strain increment can be written as:

$$\{d\epsilon\} = \{d\epsilon_e\} + \{d\epsilon_p\} \qquad A(22)$$

where the elastic strain increments are related to the stress increments by the symmetric material matrix D. The plastic strain increments are related to the yield criterion through Drucker's normality principle

$$\{d\varepsilon_p\} = \lambda \{\frac{\partial F}{\partial \sigma}\} \qquad A(23)$$

Therefore; Eq. A(22) can be rewritten as:

$$\{d\varepsilon\} = [D]^{-1} \{d\sigma\} + \lambda \{\frac{\partial F}{\partial \sigma}\} \qquad A(24)$$

At the point of incipient plasticity, the stresses are on the yield surface and the yield function is given by:

$$F(\sigma, k) = 0 \qquad A(25)$$

where K is a hardening parameter.

Differentiating A(25) results in:

$$d_F = \frac{\partial F}{\partial \sigma_1} \delta\sigma_1 + \frac{\partial F}{\partial \sigma_2} d_{\sigma 2} + \ldots + \frac{\partial F}{\partial k} dK = 0 \qquad A(26)$$

$$\text{or} \quad \{\frac{\partial F}{\partial \sigma}\} Td\sigma - A\lambda = 0 \quad . \qquad A(27)$$

Solving for A gives

$$A = -\frac{\partial F}{\partial k} dk \frac{1}{\lambda} \quad . \qquad A(28)$$

Equations A(24) and A(27) can be written in matrix form as

$$\begin{Bmatrix} d\varepsilon \\ 0 \end{Bmatrix} = \begin{bmatrix} D^{-1} & \frac{\partial F}{\partial \sigma} \\ \left(\frac{\partial F}{\partial \sigma}\right)^T & -A \end{bmatrix} \begin{Bmatrix} d\sigma \\ \lambda \end{Bmatrix} \qquad A(29)$$

The constant λ can be eliminated from Eq. A(23). The final expression which relates the stress changes in terms of imposed strain changes can be written as: $d\sigma = D^*_{ep}\, d\varepsilon$

A(30)

or

$$De^*_p = D - D\left\{\frac{\partial F}{\partial \sigma}\right\} \left\{\frac{\partial F}{\partial \sigma}\right\}^T D \left[A + \left\{\frac{\partial F}{\partial \sigma}\right\}^T D \left\{\frac{\partial F}{\partial \sigma}\right\}\right]^{-1} \qquad A(31)$$

where $\left\{\frac{\partial F}{\partial \sigma}\right\}^T = [F_x\ F_y\ F_z\ F_{xy}\ F_{yz}\ F_{xz}]$

and $F_x = \frac{3\sigma_1'}{2\,\overline{\sigma}}$, $F_y = \frac{3\sigma_2'}{2\,\overline{\sigma}}$, $F_z = \frac{3\sigma_3'}{2\,\overline{\sigma}}$

$$F_{xy} = \frac{3T_{xy}}{\overline{\sigma}}, \quad F_{yz} = \frac{3T_{yz}}{\overline{\sigma}}, \quad F_{zx} = \frac{3T_{zx}}{\overline{\sigma}} \qquad A(32)$$

in which the dashes stand for deviatoric stresses i.e.

$$\sigma_1' = \sigma_x - \frac{(\sigma_x + \sigma_y + \sigma_z)}{3} \text{ etc.}$$

The elastic-plastic matrix D^*_{ep} replaces the elastic matrix D in incremental elastic-plastic analysis. The plastic load vector for the elements which deform plastically is given by:

$$\{dq\} = \iiint [B]^T \{d\dot{\sigma}\} \, dv_m \qquad A(33)$$

where $\{d\dot{\sigma}\}$ is defined as:

$$\{d\dot{\sigma}\} = \{d\sigma_e\} - \{d\sigma\} + ([De] - [Dep]) \{d\epsilon\} \qquad A(34)$$

APPENDIX B

Description of the Finite-Element Computer Program

The computer program presented here was based on the three-dimensional 8-noded linear isoparametric cubic element. The optimum goal of this study was to develop a three-dimensional nonlinear computer program capable of extending a crack and changing the boundary conditions for the model under consideration. This program in its present form is not a general analysis program for nonlinear cracked structures. The restrictions are listed as follows: (1) the crack must lie on the x-axis and propagate in the positive x-direction, (2) the configuration and loading must be symmmetric about the x-axis.

The input to the program is illustrated by using one eighth of a center-crack panel shown in Fig. 4.

1. CRACK, WIDTH, THICK, HEIGHT, DAX:, SCALE (6E10.4)

   The format for each input is shown in parenthesis. Crack specifies the crack length in the y=0 plane. Width, thick, height represent width, thickness and height of the structure., DAX is defined as the smallest element size in the region and is used for the crack-extension in the program. Scale, scales the width, thickness and height of the specimen to the desired dimension.

2. LPRIT, LMAX, KMAX, NLAYER, NEP (16I5)

   LPRIT = 0 indicates that no intermediate output is printed. LPRIT = 1 results in intermediate output. LMAX is the number of nodes in Z=0 plane. KMAX is the number not elements in Z=0 plane. NLAYER indicates the number of layers in the structure. NEP specifies elastic or

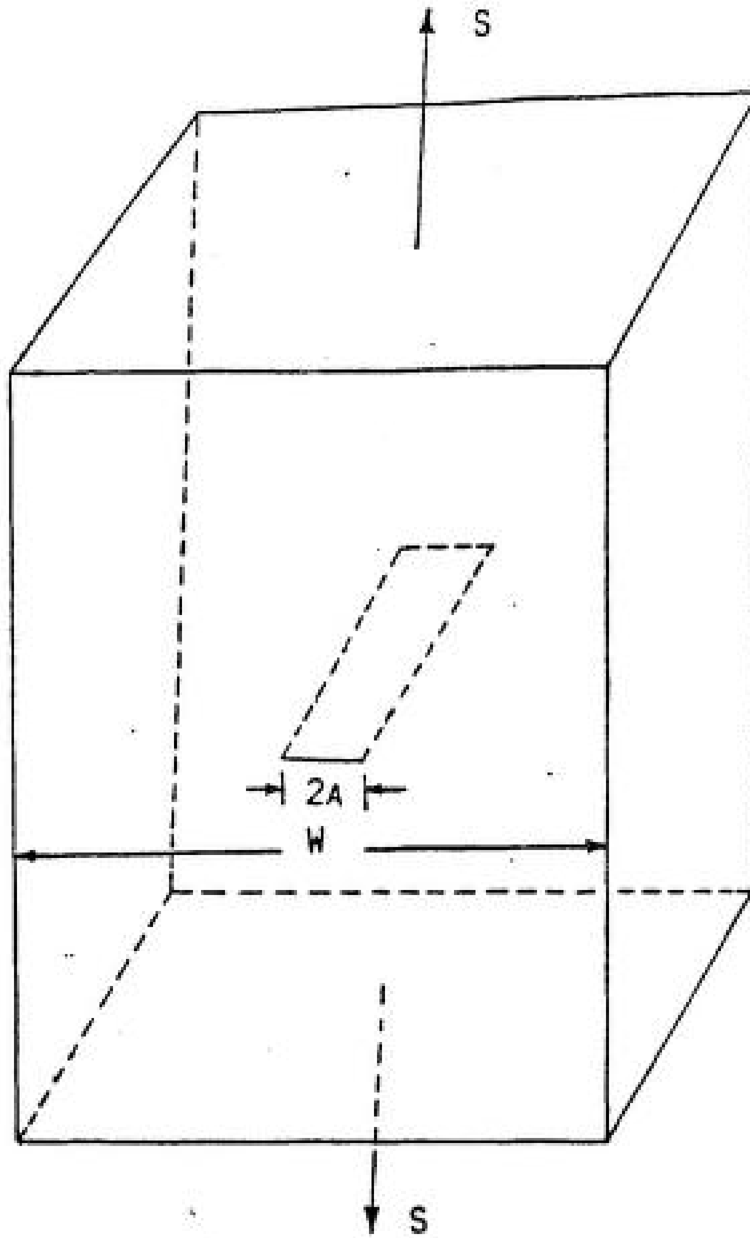


Figure 4. Center-Crack Panel subjected to uniform stress.

plastic anaslysis if NEP=0, elastic analysis is performed. If NEP > 0, the plastic analysis is performed.

3. K, XR(I), YR(I), ZR(I), (I5, 4X , 3 E15.7)

   K refers to the node number, and XR(I), YR(I), ZR(I) are the coordinates of node K in x,y and z direction, respectively.

4. IN, (MODE(J, IN), J = 1,8) (16I5)

   IN describes the element number, and node gives the nodal connective of each cubic element in the structure.

5. NSYMPL (16I5)

   NSYMPL specifies the number of symmetric planes

6. (ISYMPLY(I), I = 1, NYSMPL) (16I5)

   ISYMPL describes the corresponding numbers designated for each plane in the structure.

7. NFIX, NLOAD, SNPD (16I5)

   NFIX, NLOAD NSPD describe the number of fixed loaded, and specified displacements for nodes, respectively.

8. NODF, MU, MV, MW (16I5)

   NODF describes the number of fixed nodes, and MU, MV and Mw represents the u, v and w displacements fixed for each node.

9. Nodlod (IL), P_x, $P_{y,}$ P_z (16I5)

   Nodlod specifies the number of loaded nodes, and p_x, p_y and p_z represent the components of loading in x, y, and z direction, respectively.

10. NODS, K, DISP(N) (16I5)

   NODS is the node number, K is the code for u, v and w displacements, and Disp is the specified displacement for the corresponding node.

11. NTYP, NLM, SCRIT, RP, ACURCY (2I5, 4E10.4)

    NTYP stands for the crack growth criterion. NLM is the number of increments to release the crack tip force. SCRIT is used for the CTOD criterion. RP is the relaxation parameter and ACURCY is used for the crack opening displacement accuracy.

12. P, WORD (E103, 1X, A_4)

    P designates the maximum applied stress for each cycle. The word specifies stationary or growing crack for each cycle. If word is set equal to grow, the crack will extend one element size. If word is equal to halt, the crack will be stationary for that cycle.

APPENDIX C

FORTRAN LISTING

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```
      PROGRAM CRACK1(INPUT,OUTPUT,TAPE7=D1,TAPE5=INPUT,
     1TAPE6=OUTPUT)
      COMMON/MAIN/AA(2000000),BB(9600,1),D(6,6),DINV(6,6),
     1DISP(80),EPS(12400),EFEST(1550),FORCE(10),LINE(100),
     2LOCAT(10),LBFOR(10),MB(9600),MSUM(9600),MPTAB(9600),
     3MPLAS(12400),MODE(8,1550),MPLC(1550),NODXO(700),
     4NODYO(700),NODZO(700),NODXC(700),NODYC(700),NODZC(700),
     5NODFIX(80),NODLOD(80),NDISP(80),R(9600),
     6SIGBAR(12400),SK(24,24),T1(9600),T2(9600),T4(2000),
     7T3(9600),U(3200),UOLD(3200),V(3200),VOLD(3200),V2(3200),
     8W(3200),WOLD(3200),X(74400),XR(3200),Y(74400),
     9YR(3200),Z(9600),ZR(3200)
      COMMON/CNST/EPSI,SK2,LMAX,KMAX,DAX,
     1PYLD,SCRIT,YOUNG,POIS,CRACK,PT,WIDTH,PMAX,HP,
     2SBAR,LPRIT,NGAUS,NLAYER,NNODE,
     3INODXO,INODYO,INODZO,INODXC,INODYC,INODZC,LNSTIF,MXNOD,
     4MXNEL,MXGAUS,ICUT,LTOTB,ITNODX,KLU,NTYP,NLM,
     5NDOF,KNEW,NEP,ERIT,NELM,AM,ROM
      COMMON/MLTNMAT/YSTRS(20),YSTRN(20),PLNODR(20),NSEGMT
      COMMON/D3H2/NNPE,NDF,NQD,NSTR,NQD2,NNPE2,NQD2NPE,NQD2SR,MXQ2S
      COMMON/VECT/ STRV(8,6),STRSV(8,6),BMT(64,3),WDUM(8),XE(8,3),
     C NCUBE(8),DIS(8,3)
       DIMENSION IIMAX(3200),NSAME(3200,20),MS(8)
       DIMENSION JNEW(3200),TITLE(20),ISYMPL(6),NBEGIN(8),NEND(8)
      DIMENSION STR(6)
C      *****************************************************
C      * XR(I,J),YR(I,J) COORDINATES OF RECTANGULAR ELEMENTS*
C      *WHICH ARE LOCATED IN THE Z=0 PLANE.
C      * XR(I),YR(I),ZR(I) COORDINATES OF NODES IN THE STR*
C      *UCTURE.                                            *
C      *NODXO(I) NODE NUMBERS FOR PLANE X=0               *
C      *NODYO(I)                        Y=0               *
C      *NODZO(I)                        Z=0               *
C      *NODXC(I)                        X=XCOR            *
C      *NODYC(I)                        Y=YCOR            *
C      *NODZC(I)                        Z=ZCOR            *
C      *U(I),V(I),W(I)  DISPACEMENT COMPONENTS FOR EACH NODE IN THE SPECIMEN
C      NODLOD(80)  MAX OF 80 NODES LOADED
C      *
C     EPSI IS ACCURACY CHECK VALUE
C     SK2  STIFFNESS OF SPRINGS CONNECTED TO BOUNDARY NODES
C     LMAX  NO OF NODES IN Z=0 PLANE
C     KMAX  NO OF ELEMENTS IN Z=0 PLANE
C     DAX   SMALLEST ELEMENT SIZE IN THE STRUCTURE
C     PYLD  LOAD AT INITIAL YIELD
C     SCRIT  USED FOR CTOD  CRITERION
C     YOUNG  YOUNGS MODULUS OF THE MATERIAL
C     POIS  POISSON RATIO OF THE MATERIAL
C     CRACK   CRACK LENGTH
C     PT   VARIABLE USED FOR LOADING
C     WIDTH  WIDTH OF THE SPECIMEN
C     SIGYS  YIELD STRESS OF THE MATERIAL
C     LPRIT  LPRIT GREATER THAN 0 NO INTERNAL OUTPUT ,LPRIT=0
C     INTERNAL OUTPUT(USED FOR SMALL PROBLEMS)
C     NGAUS  NO GAUSS POINTS IN EACH DIRECTION
C     NLAYER  NO OF LAYERS PUT IN THE STRUCTURE
C     NNODE  TOTAL NO OF NODES IN THE STRUCTURE
C     INODXO  TOTAL NO OF NODES IN X=0 PLANE
C     INODYO  TOTAL NO OF NODES IN Y=0 PLANE
C     INODZO  TOTAL NO OF NODES IN Z=0 PLANE
C     INODXC  TOTAL NO OF NODES IN X=C PLANE
C     INODYC  TOTAL NO OF NODES IN Y=C PLANE
C     INODZC  TOTAL NO OF NODES IN Z=C PLANE
C     LNSTIF  MAXIMUM DIMENSION FOR AA MATRIX
C     MXNOD  MAXIMUM NODES PUT INTO THE PROGRAM
C     MXNEL  MAXIMUM ELEMENTS PUT INTO THE PROGRAM
```

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C      MXNOD AND MXNEL ARE FOR DIMENSIONAL PURPOSES
C      MXGAUS  MAXIMUM NO OF ELEMENTS MULTIPLY THE NO OF GAUSS
C      POINTS IN EACH DIRECTION(X,Y,Z IF NGAUS=2,THE NO IS 2*2*2)
C      ICUT  VARIABLE USED IN BREAK SUBPROGRAM FOR RELEASING FORCES
C      LTOTB  TOTAL NO NODES IN THE THICNESS ALONG THE CRACK TIP
C      ITNODK  TOTAL NO OF NODES ALONG THE CRACK LINE
C      KLU  VARIABLE USED FOR CRACK EXTENSION
C      NTYP VARIABLE USED FOR TYPE OF CRACK EXTENSION
C      NLM  NO OF INCREMENTS TO RELEASE THE NODAL FORCES
C      NLOAD NO OF LOADED NODES IN THE STRUCTURE
C      NSPD  NO OF SPECIFIED DISPLACEMENTS
C      MAXIT  MAX NO OF ITERATION USED FOR CONVERGENCE PURPOSES
C      NDOF  TOTAL NO OF DEGRES OF FREEDOM IN THE MODEL
C      NEP  IF NEP =0 ELASTIC ANALYSIS,IF NEP GREATER 0 PLASTIC ANAL
C      ERIT  ACCURACY CHECK VALUE FOR CONVERGENCE USED IN SUB PLAS
C      NELM  TOTAL NO OF ELEMENTS IN THE SYSTEM
C      AM,ROM  LINEAR OR NONLINEAR STRAIN HARDENING COEFFICIENTS
C      IF AM=0 MATERIAL IS ELASTIC-PERFECTLY .
C      KNEW  VARIABLE USED IN CONTACT SUBPROGRAM TO CHECK WHETHER
C      THE NODE CLOSED OR OPENED.
C
       DATA NNPE,NDF,NQD,NSTR/8,3,2,6/
C ***  OPEN MAP AND ZERO THE AA VECTOR OF LENGTH LENTOT
       LENTOT=2000000+9600*9+1550*130+700*(6)+80*4+100
      1+10*3+72+2000+3200*10+576
       J=LENTOT/65536                                                NEW
       JJ=LENTOT-(LENTOT/65536)*65536
       IF(JJ.NE.0) J=J+1                                          NEW
       LOPN=J*128                                                 NEW
       CALL OPEN(LOPN)
C ***  ZEROING THE VECTORS
       J=LENTOT/65536
       DO 223 I=1,J
       I1=(I-1)*65535+1
       AA(I1;65535)=0.0
223    CONTINUE
       J=J*65536+1
       JJ=LENTOT-J+1
       AA(J;JJ)=0.0
C
CC ***
C
           LNSTIF=2000000
       MXNEL=1550
       MXNOD=3200
       NGAUS=2
       NQD2=NGAUS**3
       NNPE2=(NNPE*(NNPE+1))/2
       NQD2NPE=NQD2*NNPE
       NQD2SR=NQD2*NSTR
       MXQ2S=MXNEL*NQD2SR
       MXGAUS=NQ2*MXNEL
       LL=3*MXNOD
       MPTAB(1;LL)=0
       Z(1;LL)=0.0
       R(1;LL)=0.0
       BB(1,1;LL)=0.0
       ZR(1;MXNOD)=0.0
       CALL Q3CLOCKS(CPU,WALL)
42     FORMAT(I5,3F10.3)
C
C ***  READ GEOMETRIC DATA
C
       READ (5,222) (TITLE(I),I=1,20)
222    FORMAT(20A4)
       WRITE(6,15) (TITLE(I),I=1,20)
```

```
15    FORMAT(1H1///5X,20A4)
      READ(5,16) CRACK,WIDTH,THICK,HEIGHT,DAX,SCALE
      WRITE(6,17) CRACK,WIDTH,THICK,HEIGHT,DAX,SCALE
16    FORMAT(6E10.4)
17    FORMAT(5X,'CRACK=',F10.4,2X,'WIDTH=',F10.4,2X,'THICK=',F10.4,
     C//5X,'HEIGHT=',F10.4,2X,'DAX=',F10.6,2X,'SCALE=',F10.5)
39    FORMAT(16I5)
      XCOR=WIDTH
      YCOR=HEIGHT
      ZCOR=THICK
      EPSI=1.E-10
      READ(5,39) LPRIT,LMAX,KMAX,NLAYER,NEP
      WRITE(6,28) LPRIT,LMAX,KMAX,NLAYER,NEP
28    FORMAT(5X,'LPR=',I2,2X,'LMAX=',I5,2X,'KMAX=',I5,2X,
     C 'NLAYER=',I2,2X,'NEP=',I2)
      NNODE=(NLAYER+1)*LMAX
      NDOF=NNODE*3
      NELM=KMAX*NLAYER
C --- CONSTANTS IN POLYNOMIAL AND D-MATRIX
      CALL ACAL
      READ (5,39) NMAT,NSEGHT
      DO 3 I=1,NMAT
      READ(5,16) YOUNG,POIS,SIGYS,AM,ROM
      WRITE(6,4) YOUNG,POIS,SIGYS,AM,ROM
      READ(5,39) (NBEGIN(IG),NEND(IG),IG=1,8)
      WRITE(6,39)(NBEGIN(IG),NEND(IG),IG=1,8)
      DO 5 IG=1,8
      IF(NBEGIN(IG).EQ.0) GOTO 3
      I1=NBEGIN(IG)*8-7
      I2=NEND(IG)*8-I1+1
 5    SIGBAR(I1;I2)=SIGYS
3     CONTINUE
4     FORMAT(//10X,'MODULUS, NUE, YIELD STRESS, AM, & ROM:',5E12.4)
      CALL DCON(YOUNG,POIS,D,DINV)
C
C *** READ            COORDINATES AND CONNECTIVITY
C
      DO 30 I=1,NNODE
      JNEW(I)=I
30    READ(7,20) K,XR(I),YR(I),ZR(I)
20    FORMAT(I5,4X,3E15.7)
      WRITE(6,333)
      WRITE(6,861)(J,XR(J),YR(J),ZR(J),J=1,NNODE)
861   FORMAT(2(3X,I5,3(E13.6,1X)))
333   FORMAT(1H1//10X,'NODAL COORDINATES,NODE#, X,Y,AND,Z'//)
      DO 31 IE=1,NELM
31    READ(7,39) IN,(NODE(J,IN),J=1,8)
      WRITE(6,334)
334   FORMAT(1H1//5X,'NODAL CONNECTIVITY IE, I,J,K,L, I1,J1,K1,L1'//)
      WRITE(6,864) (IE,(NODE(J,IE),J=1,8),IE=1,NELM)
864   FORMAT(2(5X,9I5))
C
C ***
C
       IZIP1=5
      CALL Q3CLOCKS(CPU,WALL)
      WRITE(6,9999) IZIP1,CPU,WALL
9999  FORMAT(5X,'STEP#',I3,2X,'TIME IN SECS: CPU=',F10.4,2X,
     C'WALL=',F12.3)
       WRITE(6,1607) NELM
1607   FORMAT(5X,'TOTAL NO OF HEXAHEDRAN=',I6)
C *** IDENTIFY  NODES ON CONSTANTS PLANES
C          IDENTIFY X=0 PLANE ,STORE NODXO ARRAY
C          IDENTIFY Y=0 PLANE ,STORE NODYO ARRAY
C          IDENTIFY Z=0 PLANE ,STORE NODZO ARRAY
C          IDENTIFY X=C PLANE ,STORE NODXC ARRAY
```

```
C          IDENTIFY Y=C PLANE ,STORE NODYC ARRAY
C          IDENTIFY Z=C PLANE ,STORE NODZC ARRAY
C
        INODXO=0
        INODYO=0
        INODZO=0
        INODXC=0
        INODYC=0
        INODZC=0
        DO 1300 I=1,NNODE
        IF(ABS(XR(I)).LE.EPSI) GO TO 1301
        DX1=ABS(XR(I)-XCOR)
        IF(DX1.GT.EPSI) GO TO 1302
          INODXC=INODXC+1
          NODXC(INODXC)=I
          GO TO 1302
1301    INODXO=INODXO+1
        NODXO(INODXO)=I
1302    IF(ABS(YR(I)).LE.EPSI) GO TO 1303
        DY1=ABS(YR(I)-YCOR)
        IF(DY1.GT.EPSI) GO TO 1304
          INODYC=INODYC+1
          NODYC(INODYC)=I
          GO TO 1304
1303      INODYO=INODYO+1
          NODYO(INODYO)=I
1304    IF(ABS(ZR(I)).LE.EPSI) GO TO 1305
        DZ1=ABS(ZR(I)-ZCOR)
        IF(DZ1.GT.EPSI) GO TO 1300
          INODZC=INODZC+1
          NODZC(INODZC)=I
        GO TO 1300
1305      INODZO=INODZO+1
          NODZO(INODZO)=I
1300    CONTINUE
        WRITE(6,1002)
1002    FORMAT(5X,'INODXO,5X,INODYO,5X,INODZO,5X,INODXC,5X,INODYC
     1,5X,INODZC')
        WRITE(6,1122) INODXO,INODYO,INODZO,INODXC,INODYC,INODZC
1122      FORMAT(8X,6I6)
       IF(LPRIT.EQ.0) WRITE(6,39) (NODXO(I),I=1,INODXO)
       IF(LPRIT.EQ.0) WRITE(6,39) (NODYO(I),I=1,INODYO)
       IF(LPRIT.EQ.0) WRITE(6,39) (NODZO(I),I=1,INODZO)
       IF(LPRIT.EQ.0) WRITE(6,39) (NODXC(I),I=1,INODXC)
       IF(LPRIT.EQ.0) WRITE(6,39) (NODYC(I),I=1,INODYC)
       IF(LPRIT.EQ.0) WRITE(6,39) (NODZC(I),I=1,INODZC)
1001    CONTINUE
       IZIP1=10
       CALL Q3CLOCKS(CPU,WALL)
       WRITE(6,9999) IZIP1,CPU,WALL
C
C ****
C
       CALL NSAMC(NODE,NSAME,IIMAX,MB,NNODE,NELM,8,MXNOD,MXNEL,NDOF)
        MSUM(1)=0
        MSUM(2)=1
        DO 352 I=3,NDOF
        LN=I-1
352     MSUM(I)=MSUM(LN)+MB(LN)
        LDOF=MSUM(NDOF)+MB(NDOF)
        WRITE(6,504) LDOF
504     FORMAT(//10X,'STORAGE REQUIREMENT FOR STIFFNESS MATRIX IS='I10)
        IZIP1=14
       CALL Q3CLOCKS(CPU,WALL)
       WRITE(6,9999) IZIP1,CPU,WALL
C
```

```
      SK2=YOUNG*1.0E+07
C
C     ASSEMBLE THE STIFFNESS MATRIX K
C
      DO 943 I=1,NELM
      NCUBE(1;8)=NODE(1,I;8)
      MS(1;8)=NCUBE(1;8)
      CALL CORDIN(NCUBE,MXNOD,XR,YR,ZR,XE)
      CALL SMALLK(SK,XE,D,IERR)
      DO 943 J=1,8
      DO 943 L=1,8
       IF(MS(L).LT.MS(J)) GO TO 943
      IU=3*MS(J)-2
      IV=IU+1
      IW=IV+1
      JU=3*MS(L)-2
      JV=JU+1
      JW=JV+1
      N1=MSUM(JU)-JU+MB(JU)+IU
      N2=N1+1
      N3=N2+1
      N4=MSUM(JV)-JV+MB(JV)+IU
      N5=N4+1
      N6=N5+1
      N7=MSUM(JW)-JW+MB(JW)+IU
      N8=N7+1
      N9=N8+1
      MC1=3*J-2
      MC2=MC1+1
      MC3=MC2+1
      MR1=3*L-2
      MR2=MR1+1
      MR3=MR2+1
      AA(N1)=AA(N1)+SK(MR1,MC1)
      AA(N4)=AA(N4)+SK(MR2,MC1)
      AA(N5)=AA(N5)+SK(MR2,MC2)
      AA(N7)=AA(N7)+SK(MR3,MC1)
      AA(N8)=AA(N8)+SK(MR3,MC2)
      AA(N9)=AA(N9)+SK(MR3,MC3)
952   IF(J.EQ.L) GO TO 943
      AA(N2)=AA(N2)+SK(MR1,MC2)
      AA(N3)=AA(N3)+SK(MR1,MC3)
      AA(N6)=AA(N6)+SK(MR2,MC3)
943   CONTINUE
      IZIP1=18
      CALL Q3CLOCKS(CPU WALL)
      WRITE(6,9999) IZIP1,CPU,WALL
C
C *** IMPOSE SYMMETRIC BOUNDARY CONDITIONS
C
      READ (5,39) NSYMPL
      WRITE(6,315) NSYMPL
315   FORMAT(/5X,' # OF SYMMETRIC BOUNDARY CONDITIONS =',I3)
      IF(NSYMPL.EQ.0) GOTO 314
      READ(5,39) (ISYMPL(I),I=1,NSYMPL)
      WRITE(6,316) (ISYMPL(I),I=1,NSYMPL)
316   FORMAT(10X,' SYMMETRIC PLANE NUMBERS ARE :',6I3)
      DO 317 IS=1,NSYMPL
      ISY=ISYMPL(IS)
      IF(ISY.EQ.1) CALL SYMPLN(AA,MSUM,MB,MPTAB,NODXO,INODXO,SK2,1,
     CNDOF,LNSTIF)
      IF(ISY.EQ.2) CALL SYMPLN(AA,MSUM,MB,MPTAB,NODYO,INODYO,SK2,2,
     CNDOF,LNSTIF)
      IF(ISY.EQ.3) CALL SYMPLN(AA,MSUM,MB,MPTAB,NODZO,INODZO,SK2,3,
     CNDOF,LNSTIF)
      IF(ISY.EQ.4) CALL SYMPLN(AA,MSUM,MB,MPTAB,NODXC,INODXC,SK2,1,
```

```
     CNDOF,LNSTIF)
      IF(ISY.EQ.5) CALL SYMPLN(AA,MSUM,MB,MPTAB,NODYC,INODYC,SK2,2,
     CNDOF,LNSTIF)
      IF(ISY.EQ.6) CALL SYMPLN(AA,MSUM,MB,MPTAB,NODZC,INODZC,SK2,3,
     CNDOF,LNSTIF)
317   CONTINUE
314   CONTINUE
C
C *** SYMMETRIC BOUNDARY CONDITIONS ON THE CRACK PLANE
C
       DO 318 I=1,INODYO
       L=NODYO(I)
       SAP=XR(L)
       IF(SAP.LT.CRACK) GO TO 318
       NV=3*L-1
       NVNV=MSUM(NV)+MB(NV)
       MPTAB(NV)=MV
       AA(NVNV)=AA(NVNV)+SK2
318     CONTINUE
C
       IZIP1=27
      CALL Q3CLOCKS(CPU,WALL)
      WRITE(6,9999) IZIP1,CPU,WALL
C *****  READ BOUNDRAY CONDITIONS AND LOADING
C
C *** FIXED NODES AND LOADING
C
      READ(5,39) NFIX,NLOAD,NSPD
      WRITE(6,40) NFIX,NLOAD,NSPD
40    FORMAT(//5X,'# OF NODES: FIXED=',I3,2X,'LOADED=',I3,2X,
     C 'SP. DISP=',I3//)
      IF(NFIX.EQ.0) GOTO 417
         DO 416 IFIX=1,NFIX
         READ(5,39)NODF,MU,MV,MW
      WRITE(6,39) NODF,MU,MV,MW
      NODFIX(IFIX)=JNEW(NODF)
         NU=JNEW(NODF)*3-2
         NUNU=MSUM(NU)+MB(NU)
         NVNV=MSUM(NU+1)+MB(NU+1)
         NWNW=MSUM(NU+2)+MB(NU+2)
         AA(NUNU)=AA(NUNU)+MU*SK2
         AA(NVNV)=AA(NVNV)+MV*SK2
         AA(NWNW)=AA(NWNW)+MW*SK2
         MPTAB(NU)=MU
         MPTAB(NU+1)=MV
416      MPTAB(NU+2)=MW
417      CONTINUE
      IF(NLOAD.LE.0) GOTO 739
      DO 41 IL=1,NLOAD
      READ(5,42) NODLOD(IL),PX,PY,PZ
      IZ=NODLOD(IL)
      WRITE(6,43) NODLOD(IL),PX,PY,PZ
      NODLOD(IL)=JNEW(IZ)
      IZ1=(JNEW(IZ)-1)*3+1
      BB(IZ1,1)=PX
      BB(IZ1+1,1)=PY
      BB(IZ1+2,1)=PZ
41    CONTINUE
43    FORMAT(5X,I5,3(F12.5,2X))
739   IF(NSPD.LE.0) GOTO 738
      DO 735 N=1,NSPD
      READ(5,736) NODS,K,DISP(N)
      WRITE(6,737) NODS,K,DISP(N)
      NU=(JNEW(NODS)-1)*3+K
      NDISP(N)=NU
      NUNU=MSUM(NU)+MB(NU)
```

```
      AA(NUNU)=SK2
735   BB(NU,1)=SK2*DISP(N)
736   FORMAT(2I5,E14.5)
737   FORMAT(5X,I5,2X,I1,3X,E12.5)
738   CONTINUE
       R(1;NDOF)=BB(1,1;NDOF)
       IZIP1=16
      CALL Q3CLOCKS(CPU,WALL)
      WRITE(6,9999) IZIP1,CPU,WALL
       IFAC=0
       ALP=0
       IZIP1=21
       CALL SYMBAN(LNSTIF,NDOF,MB,MSUM,AA,1,BB,IFAC,T1,IERR
     1,ALP,Z,T2,T3,T4,1)
       IZIP1=22
      CALL Q3CLOCKS(CPU,WALL)
      WRITE(6,9999) IZIP1,CPU,WALL
       IF(IERR.EQ.1) WRITE(6,415) IERR
 415   FORMAT(//10X'IERR='I2,10X'NONPOSITIVE DEFINITE MATRIX')
       IF(IERR.NE.0) STOP
C      PRINT OUT UNIT LOAD DISPLACEMENTS AND STRESSES.
C
9000   CONTINUE
      WRITE(6,425)
425    FORMAT(1H1///10X,'UNIT LOAD DISPLA4EMENTS AND STRESSES'//)
       WRITE(6,418)
 418    FORMAT(6X,4HNODE,6X,1HX,15X,1HY,13X,1HZ,13X,1HU,13X,
     1 1HV,13X,1HW/)
       DO 551 N=1,NNODE
551    IIMAX(N)=(N-1)*3+1
      U(1;NNODE)= Q8VGATHR(BB(1,1;NDOF),IIMAX(1;NNODE);U(1;NNODE))
      IIMAX(1;NNODE)=IIMAX(1;NNODE)+1
      V(1;NNODE)= Q8VGATHR(BB(1,1;NDOF),IIMAX(1;NNODE);V(1;NNODE))
      IIMAX(1;NNODE)=IIMAX(1;NNODE)+1
      W(1;NNODE)= Q8VGATHR(BB(1,1;NDOF),IIMAX(1;NNODE);W(1;NNODE))
      DO 944 IN=1,NNODE
944    WRITE(6,420) IN,XR(IN),YR(IN),ZR(IN),U(IN),V(IN),W(IN)
420   FORMAT(5X,I5,2X,3(2X,E11.5),3(2X,E12.4))
C
C ***
C
      FYLD=0.0
      WRITE(6,306)
306   FORMAT(1H1//10X,'ELASTIC STRESSES: SX, SY, SZ, AND SYZ, SZX, SXY')
307   FORMAT(5X,I6,2X,6E12.4)
      IGAUSP=0
      DO 300 IE=1,NELM
      NCUBE(1;8)=NODE(1,IE;8)
      DO 301 I=1,8
      I1=NCUBE(I)
      DIS(I,1)=U(I1)
      DIS(I,2)=V(I1)
301   DIS(I,3)=W(I1)
      CALL CORDIN(NCUBE,IDXNOD,XR,YR,ZR,XE)
      CALL STRESS(DIS,XE,D,STRV,STRSV,BMT,WDUM)

      ILOC=(IE-1)*NQD2SR+1
      X(ILOC;NQD2SR)=STRSV(1,1;NQD2SR)
      Y(ILOC;NQD2SR)=STRV(1,1;NQD2SR)
      DO 350 IG=1,NQD2
      DO 360 J=1,6
360   STR(J)=STRSV(IG,J)
      IGAUSP=IGAUSP+1
      CALL SEQU(STR,SEFF)
      SEFF=SEFF/SIGBAR(IGAUSP)
      IF(FYLD.GT.SEFF) GOTO 350
```

```
      PYLD=SEFF
      IEY=IE
      IGAUSY=IG
350   CONTINUE
      WRITE(6,307) IE,(STR(J),J=1,6)
300   CONTINUE
      PYLD=1./PYLD
      WRITE(6,305) IEY,IGAUSY,PYLD
305   FORMAT(1H1//10X,'ELEMENT#',I5,2X,'GAUSS PT=',I2,2X,'LOAD FACTOR AT
     C ','YIELD=', E12.6)
      READ(5,450) NTYP,NLM,SCRIT,RP,ACURCY
      WRITE(6,412) NTYP,SCRIT,NLM,RP,ACURCY
412   FORMAT(//9X,'CRACK GROWTH CRITERION NTYP=',I2,' AND CTOD =',E10.4,
     C//10X,'NUMBER OF INCREMENTS TO RELEASE CRACK TIP FORCE=',I2,
     C//10X,'RELAXATION PARAMETER=',F5.2,'(NORMAL)',
     C//10X,'CRACK OPENING DISPLACEMENT ACCURCY=',E12.4)
450   FORMAT(2I5,4E10.4)
      IF(NEP.EQ.0) STOP
       CALL PLAS
       IZIP1=26
9991    STOP
        END
      SUBROUTINE OPEN(LOPN)
      COMMON/MAIN/AA(2000000),BB(9600,1),D(6,6),DINV(6,6),
     1DISP(80),EPS(12400),EFEST(1550),FORCE(10),LINE(100),
     2LOCAT(10),LBFOR(10),MB(9600),MSUM(9600),MPTAB(9600),
     3MPLAS(12400),MODE(8,1550),MFLC(1550),NODXO(700),
     4NODYO(700),NODZO(700),NODXC(700),NODYC(700),NODZC(700),
     5NODFIX(80),NODLOD(80),NDISP(80),R(9600),
     6SIGBAR(12400),SK(24,24),T1(9600),T2(9600),T4(2000),
     7T3(9600),U(3200),UOLD(3200),V(3200),VOLD(3200),V2(3200),
     8W(3200),WOLD(3200),X(74400),XR(3200),Y(74400),
     9YR(3200),Z(9600),ZR(3200)
      COMMON/CNST/EPSI,SK2,LMAX,KMAX,DAX,
     1PYLD,SCRIT,YOUNG,POIS,CRACK,PT,WIDTH,PMAX,HP,
     2SBAR,LPRIT,NGAUS,NLAYER,NNODE,
     3INODXO,INODYO,INODZO,INODXC,INODYC,INODZC,LNSTIF,MXNOD,
     4MXNEL,MXGAUS,ICUT,LTOTB,ITNODX,KLU,NTYP,NLM,
     5NDOF,KNEW,NEP,ERIT,NELM,AM,ROM
C
C          THIS SUB-PROGRAM OPENS ALL THE Q3OPNMAP FILES
C          MAXIMUM LENGTH OF ANY FILE IS 5376 SMALL PAGES (DECIMAL)
C
C          TO CHANGE THE MAXIMUM LENGTH CHANGE THE DATA CARD
C          DATA LMAX /          /
C
      CHARACTER*8 FILE, WORD(8)
      DATA WORD/ 'ASTIF001', 'ASTIF002',
     Z           'ASTIF003', 'ASTIF004',
     Z           'ASTIF005', 'ASTIF006',
     Z           'ASTIF007', 'ASTIF008'  /
      DATA LMAX / 5376/
      IF(LOPN.LE. LMAX) GO TO 20
      LOPNA= LMAX
      LDUM= LOPN/LMAX
      LOPNB= LOPN-LDUM*LMAX
      DO 10 I=1,LDUM
      FILE= WORD(I)
      ISTART= LMAX*512*(I-1)+1
      CALL Q3OPNMAP (IERR, FILE, AA(ISTART), LOPNA, 1)
      PRINT 100, IERR, FILE, LOPNA
      WRITE(6, 100) IERR, FILE, LOPNA
      IF(IERR.NE.0) STOP
100     FORMAT (10X,' IERR FROM OPNMAP=',Z16,5X, ' FILE ',A8,
     Z             2X,' IS OF LENGTH ',I10,2X,' SMALL PAGES (DECIMAL)',/)
10      CONTINUE
```

```
      IF( LOPNB.EQ.0) RETURN
      ISTART= LMAX*LDUM*512+1
      FILE= WORD( LDUM+1)
      CALL Q3OPNMAP (IERR, FILE, AA(ISTART), LOPNB, 1)
      PRINT 100, IERR, FILE, LOPNB
      WRITE(6, 100) IERR, FILE, LOPNB
      IF(IERR.NE.0) STOP
      RETURN
  20      CONTINUE
      FILE= WORD(1)
      CALL Q3OPNMAP ( IERR, FILE, AA(1), LOPN, 1)
      PRINT 100, IERR, FILE, LOPN
      WRITE(6, 100) IERR, FILE, LOPN
      IF(IERR.NE.0) STOP
      RETURN
      END
      FUNCTION FNMAT(SBAR,CE,EPS,H)
      COMMON/MLTNMAT/YSTRS(20),YSTRN(20),PLMODR(20),NSEGMT
C
C --- EPST= TOTAL STRAIN
C --- EPS = PLASTIC STRAIN
C
      EPST=EPS+SBAR/CE
      DO 10 I=1,NSEGMT
10    IF(EPST.LT.YSTRN(I)) GOTO 11
11    FNMAT=PLMODR(I)*CE
      RETURN
      END
      SUBROUTINE SYMFLN(AA,MSUM,MB,MPTAB,NODP,INOD,SK2,ID,NDOF,LNSTIF)
C
C *** IMPOSING SYMMETRIC BOUNDARY CONDITIONS
C
      DIMENSION AA(LNSTIF),MSUM(NDOF),MB(NDOF),MPTAB(NDOF),NODP(INOD)
      DO 100 I=1,INOD
      L=NODP(I)
      NU=(L-1)*3+ID
      NUNU=MSUM(NU)+MB(NU)
      MPTAB(NU)=NU
100   AA(NUNU)=AA(NUNU)+SK2
      RETURN
      END
      SUBROUTINE NSAMC(MSAME,NSAME,IIMAX,MB,LMAX,KMAX,NODPEL,MXNOD,
     1MXNEL,NDOF)
      DIMENSION MSAME(NODPEL,MXNEL),NSAME(MXNOD,20),IIMAX(MXNOD),
     C MB(NDOF)
C ***************************************************************
C     MXNEL = MAXIMUM NUMBER OF ELEMENTS
C     MXNOD = MAXIMUM NURBER OF NODES
C     NODPEL = # OF NOED PER ELEMENTS
C     LMAX = # OF NOEDS IN THE PROBLEM
C     KMAX = # OF ELEMENTS IN THE PROBLEM
C    MSAME(NODPEL,IEL) = NODEL CONNECTIVITY IF IEL ELEMENT
C     NDOF = LMAX* # OF DOF PER NODE
C     MB(NDOF) = BAND WIDTHS OF ALL NDOF DEGREE-OF- FREEDOM
C
C ***************************************************************
      DO 10 IE=1,KMAX
      DO 20 J=1,NODPEL
      IK=MSAME(J,IE)
      IIMAX(IK)=IIMAX(IK)+1
20    NSAME(IK,IIMAX(IK))=IE
10    CONTINUE
C16   FORMAT(16I5)
C     ***CALCULATE MB VECTOR
      IBANDW=0
      DO 350 NODE=1,LMAX
```

```
      MAXDIF=0
      IM=IIMAX(NODE)
      DO 351 M=1,IM
      NTRI=NSAME(NODE,M)
      DO 351 L=1,NODPEL
      NUM=NSAME(L,NTRI)
      NDIFF=3*(NUM-NODE)
      IF(NDIFF.LT.MAXDIF) MAXDIF=NDIFF
  351 CONTINUE
      IF(IBANDW.LT.IABS(MAXDIF)) IBANDW=IABS(MAXDIF)
      NU=3*(NODE-1)+1
      NV=NU+1
      NW=NV+1
      MB(NU)=IABS(MAXDIF)+1
      IF(MB(NU).GT.NU) MB(NU)=NU
      MB(NV)=MB(NU)+1
      MB(NW)=MB(NV)+1
  350 CONTINUE
      IBANDW=IBANDW+3
      WRITE(6,25) IBANDW
25,   FORMAT(5X,'MAX BAND WIDTH=',I6)
      RETURN
      END
       SUBROUTINE DCON(YOUNG,POIS,D,DINV)
C
C *** 3-D D(6,6) & DINV MATRICES FOR ISOTROPIC MATERIAL
C
      DIMENSION D(6,6),DINV(6,6)
       DEL=YOUNG*(1-POIS)/((1+POIS)*(1-2*POIS))
       DEL2=POIS/(1-POIS)
       DEL3=(1-2*POIS)/(2*(1-POIS))
       D(1,1;36)=0.0
      DINV(1,1;36)=0.0
       D(1,1)=DEL
       D(1,2)=DEL*DEL2
       D(1,3)=D(1,2)
       D(2,2)=D(1,1)
       D(2,3)=D(1,3)
       D(3,3)=D(1,1)
       D(4,4)=DEL*DEL3
       D(5,5)=D(4,4)
       D(6,6)=D(5,5)
C *** INVERSE OF D-MATRIX
      DINV(1,1)=1./YOUNG
      DINV(1,2)=-POIS/YOUNG
      DINV(1,3)=DINV(1,2)
      DINV(2,2)=DINV(1,1)
      DINV(2,3)=DINV(1,2)
      DINV(3,3)=DINV(1,1)
      DINV(4,4)=        2.*(1+POIS)/YOUNG
      DINV(5,5)=DINV(4,4)
      DINV(6,6)=DINV(4,4)
       DO 5 I=1,3
       DO 5 J=I,3
       D(J,I)=D(I,J)
      DINV(J,I)=DINV(I,J)
5      CONTINUE
       RETURN
       END
       SUBROUTINE SHAPE(X,Y,Z,R)
C
C ***  SHAPE FUNCTIONS
C
       DIMENSION R(8)
       R(1)=1.
       R(2)=X
```

```
      R(3)=Y
      R(4)=Z
      R(5)=X*Y
      R(6)=Y*Z
      R(7)=Z*X
      R(8)=X*Y*Z
      RETURN
      END
      SUBROUTINE CORDIN(NCUBE,MXNOD,XR,YR,ZR,A)
C
C *** EVALUATE A(8,3) CARTESIAN COORDINATE MATRIX
C
      DIMENSION  A(8,3),NCUBE(8),XR(MXNOD),YR(MXNOD),ZR(MXNOD)
      DO 1 I=1,8
      N1=NCUBE(I)
      A(I,1)=XR(N1)
      A(I,2)=YR(N1)
1     A(I,3)=ZR(N1)
      RETURN
      END
      SUBROUTINE ACAL
      COMMON/AINV/AI(8,8)
      COMMON/GENRL/GCR(8,3)
      DIMENSION R1(8),DUM(8,1),IPIVOT(8),IWK(16),A2(8,8)
       A2(1,1;64)=0.0
      DO 1 I=1,8
      X1=GCR(I,1)
      Y1=GCR(I,2)
      Z1=GCR(I,3)
      CALL SHAPE(X1,Y1,Z1,R1)
      DO 1 J=1,8
1     A2(I,J)=R1(J)
      CALL MATINV(A2,8,8,DUM,1,0,DET)
      AI(1,1;64)=A2(1,1;64)
      RETURN
      END
      SUBROUTINE DERIVE (X,Y,Z,R)
      COMMON/AINV/AI(8,8)
      ROWWISE DN(3,8)
      DIMENSION R(3,8)
      DN(1,1;24)=0.0
C****       DN/DXI NOW
      DN(1,2)=1.0
      DN(1,5)=Y
      DN(1,7)=Z
      DN(1,8)=Y*Z
C****       DN/DETA NOW
      DN(2,3)=1.0
      DN(2,5)=X
      DN(2,6)=Z
      DN(2,8)= X*Z
C****       DN/DZETA NOW
      DN(3,4)=1.0
      DN(3,6)=Y
      DN(3,7)=X
      DN(3,8)=X*Y
      DO 10 J=1,3
      DO 10 I=1,8
      R(J,I)= Q8SDOT ( DN(J,1;8) , AI(1,I;8) )
10    CONTINUE
      RETURN
      END
      SUBROUTINE SHALLK( SMK, XE, D, IERR)
C
C THIS MODULE GENERATES AN ELEMENTAL STIFFNESS MATRIX FOR THE GIVEN
C  ELEMENT. VECTOR VERSION
```

```
C
      DIMENSION SMK(24,24), XE(8,3),D(6,6)
      COMMON/D382/NNPE,NDF,NQD,NSTR,NQD2,NNPE2,NQD2NPE,NQD2SR,MXQ2S
      REAL KE
      DIMENSION KE(324)
C
C
C  INPUT:    D(6,6) = MODULUS MATRIX
C            XE(8,3) = 8 NODES X,Y,  Z COORDINATES
C  OUTPUT :   SMK(24,24) = STIFFNESS MATRIX
C
C
C
C
C KE        - ELEMENTAL STIFFNESS MATRICES FOR ALL DISTINCT ELEMENTS, IN
C             ROWISE NODAL BLOCK LOWER TRIANGULAR FORM
C PE        - ELEMENTAL LOAD VECTORS FOR ALL ELEMENTS IN NODAL BLOCK FORM
      DATA NDFX, NSTRX, NNPEX / 3, 6, 8 /
C
C
C
C
C NDF      - NUMBER OF DISPL. DEGREES OF FREEDOM PER NODE
C NSTR     - NUMBER OF STRESS RESULTANTS PER NODE
C NQD      - NUMBER OF QUADRATURE POINTS IN EACH DIRECTION
C NNPE     - NUMBER OF NODES PER ELEMENT
C
C
C***    ETH= THERMAL STRAINS IN THE CARTESIAN SYSTEM.
C****   FTHERM= THERMAL LOAD VECTOR.
C
C
C [D] - STRESS STRAIN MATRIX
C
      DIMENSION IBSP(6,3), B(64,3), BJ(288,3),CK(288,3),
     Z           WTDETEX(64), SUM(288)
      DIMENSION WTDET(8)
      DIMENSION CTH(64), TPST(8)
      DATA IBSP/  1, 2*0, 2, 0, 3,
     Z            0, 2, 0, 1, 3, 0,
     Z            2*0, 3, 0, 2, 1 /
C
C [IBSP]    - SPARSITY PATTERN AND POINTER MATRIX FOR [B] AND [BJ]
C [B]       - STRAIN DISPLACEMENT MATRIX
C [BJ]      - ANOTHER STRAIN DISPLACEMENT MATRIX
C [CK]      - A ROW FOR EACH STIFFNESS MATRIX NODAL PARTITION
C (WTDETEX) - REPLICATED WEIGHTED DETERMINANTS
C (SUM)     - TEMPORARY STORAGE
C
      DIMENSION  IREPL(36),IPOSN(210)
      DESCRIPTOR IREPLD, SORCD, DESTD
C
C IREPL   - VECTOR OF LENGTH "NNPE2" CONTAINING ZEROS USED IN THE
C           REPLICATION PROCESS
C IPOSN   - ARRAY OF LENGTH "NNPE2" USED TO CORRECTLY POSITION
C           THE NODAL PARTITIONS IN [KE]
C IREPLD  - VECTOR DESCRIPTOR FOR (IREPL)
C SORCD   - VECTOR DESCRIPTOR FOR THE REPLICATION SOURCE
C DESTD   - VECTOR DESCRIPTOR FOR THE REPLICATION DESTINATION
C
      DATA LENI, LENB, LENBJ, LENC, LENW, LENWT
     Z    / 18, 192,  864,  864, 288,  64 /
C
C THESE ARE THE DIMENSIONED LENGTHS OF [IBSP], [B], [BJ], [CK],
C (WTDETEX), AND (SUM) FOR ZEROING OUT PURPOSES.
C
```

```
          L3 = NQD2NPE - NQD2 + 1
          DO 140 KK = 1, NNPE
            ASSIGN IREPLD, IREPL(1; KK)
            ASSIGN SORCD, B(L3,IT; NQD2)
            ASSIGN DESTD, BJ(L1,IT; L2)
            CALL Q8VXTOV(X'02', 0, IREPLD, 0, SORCD, 0, DESTD)
            L2 = L2 + NQD2
            L1 = L1 - L2
            L3 = L3 - NQD2
 140      CONTINUE
 150    CONTINUE
C
      DO 155 IT=1,3
        B(1,IT;NQD2NPE) = B(1,IT;NQD2NPE)*WTDETEX(1;NQD2NPE)
 155    CONTINUE
C
C FORM ([B]**T * [D]) * [BJ] A ROW AT A TIME.
C
      L9=1
      DO 400 II = 1, NDF
        CK(1,1; LENDF) = 0.
        DO 230 KK = 1, NSTR
          SUM(1; NQD2NPE) = 0.
          DO 210 JJ = 1, NSTR
            IT = IBSP(JJ,II)
            IF (IT .EQ. 0) GOTO 210
            IF (D(JJ,KK) .EQ. 0.) GOTO 210
              SUM(1; NQD2NPE) = SUM(1; NQD2NPE) + B(1,IT; NQD2NPE)
                                                * D(JJ,KK)
 210      CONTINUE
C
C FILL UP THE REST OF (SUM)
C
          IF (SUM(1) .EQ. 0.) GOTO 225
            L1 = LE - NQD2 + 1
            L2 = NQD2
            L3 = NQD2NPE - NQD2 + 1
            DO 215 JJ = 2, NNPE
              SUM(L1; L2) = SUM(L3; L2)
              L2 = L2 + NQD2
              L1 = L1 - L2
              L3 = L3 - NQD2
 215        CONTINUE
            DO 220 JJ = 1, NDF
              IT = IBSP(KK,JJ)
              IF (IT .EQ. 0) GOTO 220
                CK(1,JJ; LE) = CK(1,JJ; LE) + SUM(1; LE) * BJ(1,IT; LE)
 220        CONTINUE
 225      CONTINUE
 230    CONTINUE
C
C WE NOW HAVE THE II-TH ROW (BEFORE SUMMING) FOR ALL
C "NNPE2" NODAL PARTITIONS OF THE ELEMENTAL STIFFNESS MATRIX.
C
        DO 310 JJ = 1, NDF
          L1 = 1
          DO 300 KK = 1, NNPE2
            L2 = L9 + IPOSN(KK)
            KE(L2) = Q8SSUM(CK(L1,JJ; NQD2))
            L1 = L1 + NQD2
 300      CONTINUE
          L9 = L9 + 1
 310    CONTINUE
 400  CONTINUE
C
      L9=1
```

```
      Z , (B(6401),DAJ23(1))
      Z , (B(7681),DAJ31(1))
      Z , (B(8961),DAJ32(1))
      Z , (B(10241),DAJ33(1))
      Z ,(B(11521),AJ11 (1 ))
      Z ,(B(11585),AJ12 (1 ))
      Z ,(B(11649),AJ13 (1 ))
      Z ,(B(11713),AJ21 (1 ))
      Z ,(B(11777),AJ22 (1 ))
      Z ,(B(11841),AJ23 (1 ))
      Z ,(B(11905),AJ31 (1 ))
      Z ,(B(11969),AJ32 (1 ))
      Z ,(B(12033),AJ33 (1 ))
      Z ,(B(12097),AJI11(1))
       EQUIVALENCE (B(12161),AJI12(1))
      Z ,(B(12225),AJI13(1))
      Z ,(B(12289),AJI21(1))
      Z ,(B(12353),AJI22(1))
      Z ,(B(12417),AJI23(1))
      Z ,(B(12481),AJI31(1))
      Z ,(B(12545),AJI32(1))
      Z ,(B(12609),AJI33(1))
      Z ,(B(12673),DET11(1))
      Z ,(B(12737),DET12(1))
      Z ,(B(12801),DET13(1))
      Z ,(B(12865),DET21(1))
      Z ,(B(12929),DET22(1))
      Z ,(B(12993),DET23(1))
      Z ,(B(13057),DET31(1))
      Z ,(B(13121),DET32(1))
      Z ,(B(13185),DET33(1))
C      DIMENSION BJ(216,3)
C**** THE ABOVE DIMENSIONS ALLOW UPTO 4 POINT GAUSSIAN IN EACH DIRECTION
       LEN=11520
       LEN1=13248
       CALL ZEROLV(BB,LEN)
       CALL ZEROLV(B,LEN1)
       NG= N*N*N
C***    NG ARE THE TOTAL NO OF INTEGRATION POINTS
       NS=8
C****     NS= NO OF SHAPE FUNCTIONS
       NSTR=6
       NCORD=3
       NFREE=3
C***    NSTR= NO OF STRAINS. NCORD= NO OF COORDINATES  NFREE= NO OF DOF P
       MAX=NG*NSTR
       DO 10 I=1,N
       X= CORD(I,N)
       XI=(X+1.)/2.
       WI=WEIGHT(I,N)
       DO 10 J=1,N
       Y= CORD(J,N)
       ETA=(Y+1.)/2.
       WJ= WEIGHT(J,N)
       DO  10  K =1,N
       Z= CORD(K,N)
       ZI=(Z+1.0)/2.0
       WK=WEIGHT(K,N)
       CALL DERIVE( XI,ETA, ZI, R)
       II=N*N*(I-1)+N*(J-1)+K
       W(II)=WI*WJ*WK/8.0
       DO 20 IJ=1,NS
       IN=NS*(II-1)+IJ
       DNX(IN)=R(1,IJ)
       DNE(IN)=R(2,IJ)
       DNZ(IN) =R(3,IJ)
```

```
20     CONTINUE
  10      CONTINUE
C*****    NOW GENERATE THE MASTER VECTOR OF THE COORDINATES
       DO 30 J=1,NG
       NT=NS*(J-1)+1
       XX(NT;NS)=XE(1,1;NS)
       YY(NT;NS)=XE(1,2;NS)
       ZZ(NT;NS) = XE(1,3;NS)
  30      CONTINUE
       LN=NG*NS
       DAJ11(1;LN)=DNX(1;LN)*XX(1;LN)
       DAJ12(1;LN)=DNX(1;LN)*YY(1;LN)
       DAJ13(1;LN) = DNX(1;LN)* ZZ(1;LN)
       DAJ21(1;LN)=DNE(1;LN)*XX(1;LN)
       DAJ22(1;LN)=DNE(1;LN)*YY(1;LN)
       DAJ23(1;LN) = DNE(1;LN)* ZZ(1;LN)
       DAJ31(1;LN) = DNZ(1;LN)* XX(1;LN)
       DAJ32(1;LN) = DNZ(1;LN)* YY(1;LN)
       DAJ33(1;LN) = DNZ(1;LN)* ZZ(1;LN)
       DO 40 I=1,NG
       NT=NS*(I-1)+1
       AJ11(I)= Q8SSUM(DAJ11(NT;NS))
       AJ12(I)= Q8SSUM(DAJ12(NT;NS))
       AJ13(I)  =Q8SSUM(DAJ13(NT;NS))
       AJ21(I)= Q8SSUM(DAJ21(NT;NS))
       AJ22(I)= Q8SSUM(DAJ22(NT;NS))
       AJ23(I)  =Q8SSUM(DAJ23(NT;NS))
       AJ31(I)  =Q8SSUM(DAJ31(NT;NS))
       AJ32(I)  =Q8SSUM(DAJ32(NT;NS))
       AJ33(I)  =Q8SSUM(DAJ33(NT;NS))
  40      CONTINUE
       DET11(1;NG) =AJ22(1;NG)*AJ33(1;NG)-AJ32(1;NG)*AJ23(1;NG)
       DET12(1;NG) =AJ21(1;NG)*AJ33(1;NG)-AJ31(1;NG)*AJ23(1;NG)
       DET13(1;NG) =AJ21(1;NG)*AJ32(1;NG)-AJ31(1;NG)*AJ22(1;NG)
       DET21(1;NG) =AJ12(1;NG)*AJ33(1;NG)-AJ32(1;NG)*AJ13(1;NG)
       DET22(1;NG) =AJ11(1;NG)*AJ33(1;NG)-AJ31(1;NG)*AJ13(1;NG)
       DET23(1;NG) =AJ11(1;NG)*AJ32(1;NG)-AJ31(1;NG)*AJ12(1;NG)
       DET31(1;NG) =AJ12(1;NG)*AJ23(1;NG)-AJ22(1;NG)*AJ13(1;NG)
       DET32(1;NG) =AJ11(1;NG)*AJ23(1;NG)-AJ21(1;NG)*AJ13(1;NG)
       DET33(1;NG) =AJ11(1;NG)*AJ22(1;NG)-AJ21(1;NG)*AJ12(1;NG)
       DET(1;NG) =AJ11(1;NG)*DET11(1;NG)-AJ12(1;NG)*DET12(1;NG)+
      ZAJ13(1;NG)* DET13(1;NG)
       AJI11(1; NG)    =  DET11(1;NG)/DET(1;NG)
       AJI12(1; NG)    =- DET21(1;NG)/DET(1;NG)
       AJI13(1; NG)    =  DET31(1;NG)/DET(1;NG)
       AJI21(1; NG)    =-DET12 (1;NG)/DET(1;NG)
       AJI22(1; NG)    =  DET22(1;NG)/DET(1;NG)
       AJI23(1; NG)    =- DET32(1;NG)/DET(1;NG)
       AJI31(1; NG)    =  DET13(1;NG)/DET(1;NG)
       AJI32(1; NG)    =- DET23(1;NG)/DET(1;NG)
       AJI33(1; NG)    =  DET33(1;NG)/DET(1;NG)
C***    JOCOBIANS AND THEIR INVERSES ARE READY
       DO 50 J=1,NG
       NT=NS*(J-1)+1
       DSX(NT;NS)  =DNX(NT;NS)*AJI11(J)+DNE(NT;NS)*AJI12(J) +DNZ(NT;NS)*
      Z  AJI13(J)
       DSY(NT;NS)  =DNX(NT;NS)*AJI21(J) +DNE(NT;NS)*AJI22(J) +DNZ(NT;NS)*
      Z  AJI23(J)
       DSZ(NT;NS)  =DNX(NT;NS)*AJI31(J) +DNE(NT;NS)*AJI32(J) +DNZ(NT;NS)*
      Z  AJI33(J)
  50      CONTINUE
C****   CARTESIAN DERIVATIVES ARE READY
       IF(ICODE.EQ.2) GO TO 320
       I1=1
       I2=NS
       CALL Q8INTVAL (0,0,I1,0,I2,0,INVA(1;NG))
```

```
      LN= NS*NG
      DO 60 I=1,NS
      NT= NG*(I-1)+1
      DNX(NT;NG)=Q8VGATHR(DSX(I;LN),INVA(1;NG);DNX(NT;NG))
      DNE(NT;NG)=Q8VGATHR(DSY(I;LN),INVA(1;NG);DNE(NT;NG))
      DNZ(NT;NG) =Q8VGATHR(DSZ(I;LN),INVA(1;NG);DNZ(NT;NG))
   60     CONTINUE
C****   CARTESIAN DERIVATIVES ARE REORDERED SO THAT THE DERIVATIVES AT A
C***    GAUSSIAN POINTS ARE GROUPED
      LEN=NS*NG
      BJ(1,1;LEN)= DNX(1;LEN)
      BJ(1,2;LEN)= DNE(1;LEN)
      BJ(1,3;LEN)= DNZ(1;LEN)
C*****      COMPUTE THE PRODUCT OF WEIGHT AND DETRMINENTS
      WDUM(1;NG)= DET(1;NG)*W(1;NG)
      RETURN
  320     CONTINUE
      LEN=NS*NG
      BJ(1,1;LEN)= DSX(1;LEN)
      BJ(1,2;LEN)= DSY(1;LEN)
      BJ(1,3;LEN)= DSZ(1;LEN)
      WDUM(1;NG)= DET(1;NG)*W(1;NG)
      RETURN
      END
      SUBROUTINE STRESS(DIS,XE,D,STR,STRS,B,WDUM)
C
C
      COMMON/D382/NNPE,NDF,NQD,NSTR,NQD2,NNPE2,NQD2NPE,NQD2SR,MXQ2S
      DIMENSION DIS(8,3),   XE(8,3),D(6,6)
      DIMENSION             STRS(8,6),STR(8,6)
      DIMENSION  WDUM( 8), SUM(64), B( 64,3), DISP(64,3)
      DIMENSION IREPL(20),STRD(6,8)
      DESCRIPTOR IREPLD,SORCD,DESTD
C
      DIMENSION IBSP(6,3)
      DATA IBSP / 1,2*0, 2, 0, 3,
     Z              0, 2, 0, 1, 3, 0,
     Z              2*0, 3, 0, 2, 1 /
C
C
      DATA NS, NSTR, NSH, NFREE /  8,  6,  3,  3  /
C*******     NS= NUMBER OF SHAPE FUNCTIONS OR NODES ON THE ELEMENT
C*******     NSTR=  NUMBER OF STRAINS
C*******     NSH= NUMBER OF INDEPENDENT DERIVATIVES IN  THE B MATRIX
C*******     NFREE=  NUMBER OF DEGREES  OF FREEDOM PER NODE
C
C
      LEN= NSTR*NQD2
      LDISP=NQD2NPE*NSH
      IREPL(1;NQD2)=0
      STRS(1,1;LEN)=0.0
      CALL ZEROLV(DISP,LDISP)
C
C****    PICK UP THE U,V,W DISPLACEMENTS SEPARATELY
C
C****    REPLICATE THE DISPLACEMENTS NQD2 TIMES
C
      ASSIGN IREPLD,IREPL(1;NQD2)
      DO 25 KC=1,NFREE
      ASSIGN SORCD, DIS(1,KC;NS)
      ASSIGN  DESTD  , DISP(1,KC;NQD2NPE)
      CALL Q8VXTOV (X'02', 0, IREPLD, 0, SORCD, 0, DESTD)
   25     CONTINUE
C****   THE MASTER DISP VECTOR READY
C
C****    GET THE CARTESIAN DERIVATIVES AT THE NODES
```

```
      NJ=(J-1)*NQD2+1
  220 FORC(IJ)=Q8SSUM(SUM(NJ;NQD2))
  400 CONTINUE
      RETURN
      END
      SUBROUTINE MATINV(A,NMAX,N,B,MAX,M,DETERM)
      DIMENSION A(NMAX,NMAX),B(NMAX,MAX)
      DIMENSION IPIVOT(100),INDEX(100,2),PIVOT(100)
C
C       IF M=0 IT CALCULATES THE INVERSE ONLY.
C       IF M=1 IT CALCULATES THE SOL TO AX=B IN B
C      INITIALIZATION
C
  10   DETERM=1.0
   15  DO 20 J=1,N
  20  IPIVOT(J)=0
  30   DO 550 I=1,N
C
C     SEARCH FOR THE PIVOT ELEMENT
C
  40  AMAX=0.0
  45   DO 105 J=1,N
  50  IF(IPIVOT(J)-1)60,105,60
  60   DO 100 K=1,N
  70   IF(IPIVOT(K)-1)80,100,740
  80  IF( ABS(AMAX)-  ABS(A(J,K)))85,100,100
  85   IROW= J
  90  ICOLUM=K
   95 AMAX= A(J,K)
  100  CONTINUE
  105  CONTINUE
  110  IPIVOT(ICOLUM)=IPIVOT(ICOLUM)+1
C
C   INTERCHANGE ROWS TO PUT ELELMENG ON DIAGONAL
C
  130  IF(IROW-ICOLUM)140,260,140
  140  DETERM= -DETERM
  150  DO 200 L=1,N
  160  SWAP= A(IROW,L)
  170  A(IROW,L)=A(ICOLUM,L)
  200  A(ICOLUM,L)= SWAP
  205 IF(M)260,260,210
  210  DO 250 L=1,M
  220  SWAP= B(IROW,L)
  230  B(IROW,L)= B(ICOLUM,L)
  250  B(ICOLUM,L)= SWAP
  260  INDEX(I,1)= IROW
  270  INDEX(I,2)= ICOLUM
  310  PIVOT(I)= A(ICOLUM,ICOLUM)
 320  DETERM= DETERM*PIVOT(I)
C
C       DIVIDE PIVOT BY PIVOT ELEMENT
C
  330  A(ICOLUM,ICOLUM)=1.0
  340  DO 350 L=1,N
  350  A(ICOLUM,L)= A(ICOLUM,L)/PIVOT(I)
  355  IF(M) 380,380,360
  360  DO 370 L=1,M
  370  B(ICOLUM,L)= B(ICOLUM,L)/PIVOT(I)
C
C   REDUCE NON-PIVOT ROWS
C
  380  DO 550L1=1,N
  390  IF(L1-ICOLUM)400,550,400
  400 T= A(L1,ICOLUM)
  420 A(L1,ICOLUM)=0.0
```

```
      LN= NS*NG
      DO 60 I=1,NS
      NT= NG*(I-1)+1
      DNX(NT;NG)=Q8VGATHR(DSX(I;LN),INVA(1;NG);DNX(NT;NG))
      DNE(NT;NG)=Q8VGATHR(DSY(I;LN),INVA(1;NG);DNE(NT;NG))
      DNZ(NT;NG) =Q8VGATHR(DSZ(I;LN),INVA(1;NG);DNZ(NT;NG))
   60     CONTINUE
C****   CARTESIAN DERIVATIVES ARE REORDERED SO THAT THE DERIVATIVES AT A
C***   GAUSSIAN POINTS ARE GROUPED
      LEN=NS*NG
      BJ(1,1;LEN)= DNX(1;LEN)
      BJ(1,2;LEN)= DNE(1;LEN)
      BJ(1,3;LEN)= DNZ(1;LEN)
C*****      COMPUTE THE PRODUCT OF WEIGHT AND DETRMINENTS
      WDUM(1;NG)= DET(1;NG)*W(1;NG)
      RETURN
  320      CONTINUE
      LEN=NS*NG
      BJ(1,1;LEN)= DSX(1;LEN)
      BJ(1,2;LEN)= DSY(1;LEN)
      BJ(1,3;LEN)= DSZ(1;LEN)
      WDUM(1;NG)= DET(1;NG)*W(1;NG)
      RETURN
      END
      SUBROUTINE STRESS(DIS,XE,D,STR,STRS,B,WDUM)
C
C
      COMMON/D382/NNPE,NDF,NQD,NSTR,NQD2,NNPE2,NQD2NPE,NQD2SR,MXQ2S
      DIMENSION DIS(8,3),   XE(8,3),D(6,6)
      DIMENSION              STRS(8,6),STR(8,6)
      DIMENSION  WDUM( 8), SUM(64), B( 64,3), DISP(64,3)
      DIMENSION IREPL(20),STRD(6,8)
      DESCRIPTOR IREPLD,SORCD,DESTD
C
      DIMENSION IBSP(6,3)
      DATA IBSP / 1,2*0, 2, 0, 3,
     Z            0, 2, 0, 1, 3, 0,
     Z            2*0, 3, 0, 2, 1 /
C
C
      DATA NS, NSTR, NSH, NFREE /  8,  6,  3,  3  /
C*******     NS= NUMBER OF SHAPE FUNCTIONS OR NODES ON THE ELEMENT
C*******     NSTR=  NUMBER OF STRAINS
C*******     NSH= NUMBER OF INDEPENDENT DERIVATIVES IN  THE B MATRIX
C*******     NFREE=  NUMBER OF DEGREES  OF FREEDOM PER NODE
C
C
      LEN= NSTR*NQD2
      LDISP=NQD2NPE*NSH
      IREPL(1;NQD2)=0
      STRS(1,1;LEN)=0.0
      CALL ZEROLV(DISP,LDISP)
C
C****    PICK UP THE U,V,W DISPLACEMENTS SEPARATELY
C
C****    REPLICATE THE DISPLACEMENTS NQD2 TIMES
C
      ASSIGN IREPLD,IREPL(1;NQD2)
      DO 25 KC=1,NFREE
      ASSIGN SORCD, DIS(1,KC;NS)
      ASSIGN  DESTD , DISP(1,KC;NQD2NPE)
      CALL Q8VXTOV (X'02', 0, IREPLD, 0, SORCD, 0, DESTD)
   25   CONTINUE
C****   THE MASTER DISP VECTOR READY
C
C****    GET THE CARTESIAN DERIVATIVES AT THE NODES
```

```
      CALL CDER( XE,NQD, B, WDUM, 2)
C
C******      NOW DO THE PRODUCT D * B* DISPLACEMENTS
      DO 100 I=1,NSTR
      SUM(1;NQD2NPE)=0.0
      DO 110 J=1,NSH
      IT= IBSP(I,J)
      IF(IT.EQ.0) GO TO 110
      SUM(1;NQD2NPE)= SUM(1;NQD2NPE)+B(1,IT;NQD2NPE)* DISP(1,J;NQ1   PE)
  110     CONTINUE
      DO 120 J=1,NQD2
      I1=(J-1)*NS+1
      STR(J,I)= Q8SSUM(SUM(I1;NS))
      STRD(I,J)=STR(J,I)
  120    CONTINUE
  100    CONTINUE
C
      DO 130 I=1,NQD2
      DO 140 J=1,NSTR
      STRS(I,J)= Q8SDOT (D(1,J;NSTR),STRD(1,I;NSTR))
  140   CONTINUE
  130   CONTINUE
      RETURN
      END
      SUBROUTINE FORCEP(BB,WTDET,STRS,FORC)
      COMMON/D382/NNPE,NDF,NQD,NSTR,NQD2,NNPE2,NQD2NPE,NQD2SR,MXQ2
       DIMENSION IBSP(6,3),B(64,3),WTDET(8),STRS(8,6)
      DIMENSION WTDETEX(64),SUM(64),SIG(64,6),IREPL(20)
      DIMENSION  FORC(24),BB(64,3),INDX(8),SH(8)
      DATA IBSP / 1,2*0, 2, 0, 3,
     Z           0, 2, 0, 1, 3, 0,
     Z           2*0, 3, 0, 2, 1 /
C
C     NQD2=NQD*NQD*NQD
C     NQD2NPE=NQD2*NNPE
      DESCRIPTOR IREPLD, SORCD, DESTD
      DESCRIPTOR BDESC
      IREPL(1;NNPE)=0
      ASSIGN IREPLD, IREPL(1; NNPE)
      ASSIGN SORCD, WTDET(1; NQD2)
      ASSIGN DESTD, WTDETEX(1; NQD2NPE)
      CALL Q8VXTOV(X'02', 0, IREPLD, 0, SORCD, 0, DESTD)
      DO 155 IT=1,NDF
      DO 156 J=1,NNPE
      DO 150 II=1,NQD2
150   INDX(II)=(II-1)*NNPE+J
      SH(1;NQD2)=Q8VGATHR(BB(1,IT;NQD2NPE),INDX(1;NQD2);SH(1;NQD2)
      I1=(J-1)*NQD2+1
      B(I1,IT;NQD2)=SH(1;NQD2)
156   CONTINUE
         BB(1,IT;NQD2NPE) = B(1,IT;NQD2NPE)*WTDETEX(1;NQD2NPE)
  155     CONTINUE
       DO 205 IS=1,NSTR
       DO 205 II=1,NNPE
       NQ=(II-1)*NQD2+1
  205  SIG(NQ,IS;NQD2)=STRS(1,IS;NQD2)
       DO 400 II=1,NDF
       SUM(1;NQD2NPE)=0.
       DO 210 JJ=1,NSTR
       IT=IBSP(JJ,II)
  213  IF (IT.EQ.0) GO TO 210
       ASSIGN BDESC ,BB(1,IT;NQD2NPE)
       SUM(1;NQD2NPE)=SUM(1;NQD2NPE)+BDESC*SIG(1,JJ;NQD2NPE)
  210  CONTINUE
       DO 220 J=1,NNPE
       IJ=(J-1)*3+II
```

```
      NJ=(J-1)*NQD2+1
  220 FORC(LJ)=Q8SSUM(SUM(NJ;NQD2))
  400 CONTINUE
      RETURN
      END
     SUBROUTINE MATINV(A,NMAX,N,B,MAX,M,DETERM)
     DIMENSION A(NMAX,NMAX),B(NMAX,MAX)
     DIMENSION IPIVOT(100),INDEX(100,2),PIVOT(100)
C
C     IF M=0 IT CALCULATES THE INVERSE ONLY.
C     IF M=1 IT CALCULATES THE SOL TO AX=B IN B
C    INITIALIZATION
C
  10  DETERM=1.0
   15 DO 20 J=1,N
  20 IPIVOT(J)=0
  30  DO 550 I=1,N
C
C   SEARCH FOR THE PIVOT ELEMENT
C
  40 AMAX=0.0
  45  DO 105 J=1,N
  50 IF(IPIVOT(J)-1)60,105,60
  60  DO 100 K=1,N
  70  IF(IPIVOT(K)-1)80,100,740
  80 IF( ABS(AMAX)-  ABS(A(J,K)))85,100,100
  85  IROW= J
  90 ICOLUM=K
   95 AMAX= A(J,K)
  100 CONTINUE
  105 CONTINUE
  110 IPIVOT(ICOLUM)=IPIVOT(ICOLUM)+1
C
C  INTERCHANGE ROWS TO PUT ELELMENG ON DIAGONAL
C
  130 IF(IROW-ICOLUM)140,260,140
  140 DETERM= -DETERM
  150 DO 200 L=1,N
  160 SWAP= A(IROW,L)
  170 A(IROW,L)=A(ICOLUM,L)
  200 A(ICOLUM,L)= SWAP
  205 IF(M)260,260,210
  210 DO 250 L=1,M
  220 SWAP= B(IROW,L)
  230 B(IROW,L)= B(ICOLUM,L)
  250 B(ICOLUM,L)= SWAP
  260 INDEX(I,1)= IROW
  270 INDEX(I,2)= ICOLUM
  310 PIVOT(I)= A(ICOLUM,ICOLUM)
 320 DETERM= DETERM*PIVOT(I)
C
C     DIVIDE PIVOT BY PIVOT ELEMENT
C
  330 A(ICOLUM,ICOLUM)=1.0
  340 DO 350 L=1,N
  350 A(ICOLUM,L)= A(ICOLUM,L)/PIVOT(I)
  355 IF(M) 380,380,360
  360 DO 370 L=1,M
  370 B(ICOLUM,L)= B(ICOLUM,L)/PIVOT(I)
C
C  REDUCE NON-PIVOT ROWS
C
  380 DO 550L1=1,N
  390 IF(L1-ICOLUM)400,550,400
  400 T= A(L1,ICOLUM)
  420 A(L1,ICOLUM)=0.0
```

```
  430  DO 450 L=1,N
  450  A(L1,L)= A(L1,L)-A(ICOLUM,L)*T
  455 IF(M) 550,550,460
 460  DO 500 L=1,M
  500  B(L1,L)= B(L1,L)-B(ICOLUM,L)*T
  550  CONTINUE
C
C   INTERCHANGE COLUMNS
C
  600  DO 710 I=1,N
  610  L=N+1-I
  620  IF(INDEX(L,1)-INDEX(L,2))630,710,630
  630  JROW= INDEX(L,1)
  640  JCOLUM= INDEX(L,2)
  650  DO 705 K=1,N
  660  SWAP= A(K,JROW)
  670 A(K,JROW)= A(K,JCOLUM)
  700  A(K,JCOLUM)= SWAP
  705  CONTINUE
  710  CONTINUE
  740  RETURN
      END

      BLOCK DATA
      COMMON/GAUSS/ CORD(8,8),WEIGHT(8,8)
       COMMON/GENRL/GCR(8,3)
      DATA CORD/ 8*0.0,
     A -0.577350269189626,0.577350269189626,6*0.0,
     B -0.774596669241483,0.0,0.774596669241483,5*0.0,
     C -0.861136311594053,-.339981043584856,0.339981043584856,
     1 0.861136311594053,4*0.0,
     D=0.906179845938664,-0.538469310105683,0.0,0.538469310105683,
     1 0.906179845938664,3*0.0,
     E -0.932469514203152,-0.661209386466265,-0.238619186083197,
     1 +0.238619186083197,0.661209386466265,0.932469514203152,2*0.0,
     F -0.949107912342759, -0.741531185599394,-0.405845151377397,0.0,
     1  0.405845151377397, 0.741531185599394,0.949107912342759,0.0,
     G  -0.960289856497536,-0.796666477413627,-0.525532409916329,
     1  -0.183434642495650,0.183434642495650,0.525532409916329,
     2   0.796666477413627,0.960289856497536/
      DATA WEIGHT /8*0.0,
     A 1.0,1.0, 6*0.0,
     B 0.555555555555556,0.888888888888889,0.555555555555556,5*0.0,
     C 0.347854845137454,0.652145154862546,0.652145154862546,
     1 0.347854845137454,4*0.0,
     D 0.236926885056189, 0.478628670499366,0.568888888888889,
     1 0.478628670499366, 0.236926885056189,3*0.0,
     E 0.171324492379170,0.360761573048139,0.467913934572691,
     1 0.467913934572691,0.360761573048139,0.171324492379170,2*0.0 ,
     F 0.129484966168870,0.279705391489277,0.381830050505119,
     1 0.417959183673469,0.381830050505119,0.279705391489277,
     2 0.129484966168870 ,0.0,
     G 0.101228536290376,0.222381034453374,0.313706645877887,
     1 0.362683783378362,0.362683783378362,0.313706645877887,
     2 0.222381034453374,0.101228536290376/
       DATA GCR/0.0,0.0,1.0,1.0,0.0,0.0,1.0,1.0,
     1           1.0,0.0,0.0,1.0,1.0,0.0,0.0,1.0,
     2           4*0.0,4*1.0/
      END
       SUBROUTINE VON(STR,SBAR,PFS)
C
C *** COMPUTE FLOW VECTOR
C
       DIMENSION PFS(6),STR(6)
       S1=2*SBAR
       PFS(1)=(2*STR(1)-STR(2)-STR(3))/S1
```

```
      PFS(2)=(2*STR(2)-STR(1)-STR(3))/S1
      PFS(3)=(2*STR(3)-STR(1)-STR(2))/S1
      PFS(4)=(6*STR(4))/S1
      PFS(5)=(6*STR(5))/S1
      PFS(6)=(6*STR(6))/S1
      RETURN
      END
      SUBROUTINE DEPL(STR,SBAR,D,HP,DPL)
      DIMENSION STR(6),STA(6),D(6,6),DD(6),DPL(6,6)
C
C ***   DEP - ELASTIC-PLASTIC MATRIX
C
      CALL VON(STR,SBAR,STA)
      B1=D(1,1)
      B2=D(4,4)
      B3=D(1,2)
      DD(1)=B1*STA(1)+B3*(STA(2)+STA(3))
      DD(2)=B1*STA(2)+B3*(STA(1)+STA(3))
      DD(3)=B1*STA(3)+B3*(STA(1)+STA(2))
      DD(4)=B2*STA(4)
      DD(5)=B2*STA(5)
      DD(6)=B2*STA(6)
      SD=B1*(STA(1)**2+STA(2)**2+STA(3)**2)+2*B3*(STA(1)*STA(2)+
     C STA(2)*STA(3)+STA(3)*STA(1))+B2*(STA(4)**2+STA(5)**2+
     C STA(6)**2)
      SD=1.0/(SD+HP)
      DO 10 I=1,6
      DO 10 J=1,6
10    DPL(I,J)=D(I,J)-DD(I)*DD(J)*SD
      RETURN
      END
        SUBROUTINE SEQU(XT,XXZ)
C       VON MISES YIELD CRITERION.
        DIMENSION XT(6)
        S1=0.5*(XT(1)-XT(2))**2
        S2=0.5*(XT(2)-XT(3))**2
        S3=0.5*(XT(3)-XT(1))**2
        S4=3*(XT(4)**2)
        S5=3*(XT(5)**2)
        S6=3*(XT(6)**2)
        ST=S1+S2+S3+S4+S5+S6
        XXZ=SQRT(ST)
        RETURN
        END
      SUBROUTINE MULTYS(A,B,N,M,C)
      DIMENSION A(N,M),B(M),C(N)
      DO 10 I=1,N
      C(I)=0.0
      DO 10 J=1,M
10    C(I)=C(I)+A(I,J)*B(J)
      RETURN
      END
       SUBROUTINE PLAS
      COMMON/MAIN/AA(2000000),BB(9600,1),D(6,6),DINV(6,6),
     1DISP(80),EPS(12400),EFEST(1550),FORCE(10),LINE(100),
     2LOCAT(10),LBFOR(10),MB(9600),MSUM(9600),MPTAB(9600),
     3MPLAS(12400),MODE(8,1550),MPLC(1550),NODXO(700),
     4NODYO(700),NODZO(700),NODXC(700),NODYC(700),NODZC(700),
     5NODFIX(80),NODLOD(80),NDISP(80),R(9600),
     6SIGBAR(12400),SK(24,24),T1(9600),T2(9600),T4(2000),
     7T3(9600),U(3200),UOLD(3200),V(3200),VOLD(3200),V2(3200),
     8W(3200),WOLD(3200),X(74400),XR(3200),Y(74400),
     9YR(3200),Z(9600),ZR(3200)
      COMMON/CNST/EPSI,SK2,LMAX,KMAX,DAX,
     1PYLD,SCRIT,YOUNG,POIS,CRACK,PT,WIDTH,PMAX,HP,
     2SBAR,LPRIT,NGAUS,NLAYER,NNODE,
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     3INODXO,INOD:O,INODZO,INODXC,INODYC,INODZC,LNSTIF,MXNOD,
     4MXNEL,MXGAUS,ICUT,LTOTB,ITNODX,KLU,NTYP,NLM,
     5NDOF,KNEW,NEP,ERIT,NELM,AM,ROM
C
      COMMON/MLTNMAT/YSTRS(20),YSTRN(20),PLMODR(20),NSEGHT
      COMMON/D382/NNPE,NDF,NQD,NSTR,NQD2,NNPE2,NQD2NPE,NQD2SR,MXQ2S
      COMMON/VECT/ STRV(8,6),STRSV(8,6),BMT(64,3),WDUM(8),XE(8,3),
     C NCUBE(8),DIS(8,3)
       DIMENSION  DPL(6,6),AMAT(8,3),STGAS(6)
       DIMENSION QP(9600),STGASV(8,6),FFTR(40)
       DIMENSION STR(6),U2(3200),INDX(3200)
       DIMENSION YTR(6),ST1(6),DPSTRN(6),STREP(6)
       DIMENSION FLV(24)
       DATA HALT,GROW/4HHALT,4HGROW/
1000   FORMAT(5X,'STEP#',I2,2X,'TIME:CPU=',F12.4,2X,
     1 'WALL=',F12.3)
C
C *** INCREMENT DISPLACEMENTS, FORCES, STRESS & STRAINS TO IST YIELD LOAD
C
      UOLD(1;NNODE)=U(1;NNODE)*PYLD
      VOLD(1;NNODE)=V(1;NNODE)*PYLD
      WOLD(1;NNODE)=W(1;NNODE)*PYLD
      R(1;NDOF)=R(1;NDOF)*PYLD
      X(1;MXQ2S)=X(1;MXQ2S)*PYLD
      Y(1;MXQ2S)=Y(1;MXQ2S)*PYLD
C *** ZEROING
      QP(1;NDOF)=0.0
      EPS(1;MXGAUS)=0.0
      MPLAS(1;MXGAUS)=0
C
C ***      READ DATA
C
      READ(5,11) PCT,ERIT,MAXIT,NODE1,NODE2,NELE1,NELE2              PLAS
   11 FORMAT(2E10.3,5I5)                                             PLAS
      READ(5,312) P,WORD
312   FORMAT(E10.3,1X,A4)
      WRITE(6,313) P,PCT,ERIT,MAXIT,NODE1,NODE2,NELE1,NELE2
313   FORMAT(///10X,'TOTAL LOAD FACTOR=',F10.4
     1/10X,'INCREMENTAL LOAD FACTOR=',F10.4/10X,'ALLOWABLE ERROR ON STRE
     2SS=',F10.4/10X,'MAXIMUM NUMBER OF ITERATION=',I4
     3/10X,'PRINT DISPLACEMENTS AT NODES',I5,' TO',I5
     4/10X,'PRINT STRESSES IN ELEMENTS',I5,' TO',I5)
      CALL Q3CLOCKS(CPU,WALL)
      IZIP1=0
      WRITE(6,1000) IZIP1,CPU,WALL
       PT=PYLD
       CALL PLOUT(NODE1,NODE2,NELE1,NELE2)
      CALL Q3CLOCKS(CPU,WALL)
      IZIP1=1
      WRITE(6,1000) IZIP1,CPU,WALL
      IF(NEP.EQ.0) STOP
       DELP=PCT*PYLD
       NPL=0
20      PMAX=PT
       PT=PT+DELP
       NPLOT=0
       IF(PT.GE.P.AND.DELP.GT.0.0) GO TO 25
       IF(PT.LE.P.AND.DELP.LT.0.0) GO TO 25
       GO TO 26
25     PT=P
       NPLOT=1
26     DELPO=PT-PMAX
       NPL=NPL+1
       KLU=0
       ICON=0
       V2(1;NNODE)=VOLD(1;NNODE)
```

```
C
C      HOLDING APPLIED LOAD CONSTANT -ITERATE UNTIL SOLUTION CONVERGE
C
       PTOY=PT/PYLD
       NBREAK=0
       GO TO 45
35     NL=-1
36     NL=NL+1
       NPLOT=0
       IF(NL.GT.NLM) GO TO 91
       ANL=NL
       DO 133 JIS=1,ICUT
       FFTR(JIS)=FORCE(JIS)*(1.-ANL/NLM)
133    WRITE(6,167)FFTR(JIS),NL
167    FORMAT(10X,'CRACK-TIP FORCE=',E16.7,'AT STEP',I2)
45     DO 50 ITER=1,MAXIT
       NC=0
65     BB(1,1;NDOF)=QP(1;NDOF)+R(1;NDOF)*PTOY
       IF(KLU.EQ.1) GO TO 830
       GO TO 831
830    DO 134 JIT=1,ICUT
       NOM=LOCAT(JIT)
       NFL=3*NOM-1
134    BB(NFL,1)=BB(NFL,1)+FFTR(JIT)
831    CONTINUE
       IFAC=1
         NC=1
        CALL SYMBAN(LNSTIF,NDOF,NB,MSUM,AA,1,BB,IFAC,T1,IERR,
     1  ALP,Z,T2,T3,T4,NC)
       DO 70 N=1,NNODE
70    INDX(N)=(N-1)*3+1
      U2(1;NNODE)= Q8VGATHR(BB(1,1;NDOF),INDX(1;NNODE);U2(1;NNODE))
      U(1;NNODE)=U2(1;NNODE)-UOLD(1;NNODE)
      UOLD(1;NNODE)=U2(1;NNODE)
      INDX(1;NNODE)=INDX(1;NNODE)+1
      U2(1;NNODE)= Q8VGATHR(BB(1,1;NDOF),INDX(1;NNODE);U2(1;NNODE))
      V(1;NNODE)=U2(1;NNODE)-VOLD(1;NNODE)
      VOLD(1;NNODE)=U2(1;NNODE)
      INDX(1;NNODE)=INDX(1;NNODE)+1
      U2(1;NNODE)= Q8VGATHR(BB(1,1;NDOF),INDX(1;NNODE);U2(1;NNODE))
      W(1;NNODE)=U2(1;NNODE)-WOLD(1;NNODE)
      WOLD(1;NNODE)=U2(1;NNODE)
C
C      COMPUTE TOTAL STRAIN INCREMENTS FROM DISPLACEMENT INCEMENTS.
C      COMPUTE ELASTIC STRESS INCREMENTS AND ADD TO CURRENT STRESSES.
C      CHECK YIELD CONDITION FOR PLASTIC ELEMENTS.
C
      IGAUSP=0
       DO 80 I=1,NELM
       DO 75 J=1,8
       NCUBE(J)=NODE(J,I)
       N1=NCUBE(J)
       DIS(J,1)=U(N1)
       DIS(J,2)=V(N1)
75     DIS(J,3)=W(N1)
C ***
      CALL CORDIN(NCUBE,MXNOD,XR,YR,ZR,XE)
      CALL STRESS(DIS,XE,D,STRV,STRSV,BMT,WDUM)
      STGASV(1,1;NQD2SR)=0.0
      ILOC=(I-1)*NQD2SR
      DO 76 IG=1,NQD2
      IGAUSP=IGAUSP+1
      SBAR=SIGBAR(IGAUSP)
      DO 77 JS=1,6
      STR(JS)=STRSV(IG,JS)
      YTR(JS)=STRV(IG,JS)
```

```
      JS1=ILOC+IG+NQD2*(JS-1)
77    ST1(JS)=X(JS1)
       CALL SEQU(ST1,S1)
        A11=STR(1)
        A22=STR(2)
        A33=STR(3)
        A44=STR(4)
        A55=STR(5)
        A66=STR(6)
        S11=ST1(1)
        S22=ST1(2)
        S33=ST1(3)
        S44=ST1(4)
        S55=ST1(5)
        S66=ST1(6)
      ST1(1;6)=ST1(1;6)+STR(1;6)
       CALL SEQU(ST1,S2)
C *** IF SOL CONVERGED THEN GOTO 801
       IF(ICON.EQ.1) GO TO 801
       IF(S2.LT.S1) MPLAS(IGAUSP)=0
       IF(S2.LT.S1) GO TO 801
       IF(MPLAS(IGAUSP).NE.0.AND.ITER.GT.1) GO TO 74
       IF(S2.LE.SBAR) GO TO 801
       MPLAS(IGAUSP)=IGAUSP
74     HP=1
C      CHECK FOR CONVERGENCE.
       IF(ABS(S2-SBAR).GT.ERIT) MC=1
       A=A11**2+A22**2+A33**2+3*(A44**2)+3*(A55**2)+3*(A66**2)
     1 -(A11*A22)-(A22*A33)-(A33*A11)
       B22=S11*(2*A11-A22-A33)+S22*(2*A22-A11-A33)+S33*(2*A33-A22-A11)
     1 +6*S44*A44+6*S55*A55+6*S66*A66
       C=S1**2-SBAR**2
         IF(A.LT.EPSI) GO TO 8
         IF(ITER.EQ.2) GO TO 200
       DELTA=B22**2-4*A*C
       IF(DELTA)200,40,40
200     PX=(SBAR-S1)/(S2-S1)
         GO TO 231
40     PONE=(-B22+SQRT(DELTA))/(2.*A)
       PTWO=(-B22-SQRT(DELTA))/(2.*A)
        PX=PONE
       IF(ABS(PONE).GT.ABS(PTWO))PX=PTWO
231      CONTINUE
        PXD=1.-PX
      YTR(1;6)=YTR(1;6)*PXD
      STR(1;6)=STR(1;6)*PXD
      IF(ROM.LE.0. .AND. NSEGMT.EQ.0) HP=AM*YOUNG
      IF(ROM.LE.0. .AND. NSEGMT.GT.0) HP=FMMAT(SBAR,YOUNG,EPS(IGAUSP),
     CIGAUSP)
      IF(ROM.GT.0.) HP=ROM**AM*SBAR**(1.-AM)/AM
      CALL DEPL(ST1,SBAR,D,HP,DPL)
      CALL MULTYS(DPL,YTR,6,6,STREP)
      CALL MULTYS(DINV,STREP,6,6,DPSTRN)
      DPSTRN(1;6)=YTR(1;6)-DPSTRN(1;6)
      STGAS(1;6)=STR(1;6)-STREP(1;6)
      CALL ERTA(DPSTRN,SMA)
      EPS(IGAUSP)=EPS(IGAUSP)+SMA
      HC=1.0
      SIGBAR(IGAUSP)=SBAR+HC*HP*SMA
       DO 12 IP=1,6
12     STGASV(IG,IP)=STGAS(IP)
8      CONTINUE
C     IF(I.EQ.1) WRITE(6,400) SBAR,EPS(IGAUSP),SIGBAR(IGAUSP),HP,S1,S2
C400  FORMAT(5X,'CONSTANTS: ',6F12.3)
76    CONTINUE
      STRSV(1,1;NQD2SR)=STRSV(1,1;NQD2SR)-STGASV(1,1;NQD2SR)
```

```
      CALL FORCEP(BMT,WDUM,STGASV,PLV)
       DO 455 IT=1,8
       IX=3*IT-2
       N1=NCUBE(IT)
       NU=3*N1-2
       QP(NU)   =QP(NU)+PLV(IX)
       QP(NU+1)   =QP(NU+1)+PLV(IX+1)
455     QP(NU+2)=QP(NU+2)+PLV(IX+2)
801   ILOC1=ILOC+1
      X(ILOC1;NQD2SR)=X(ILOC1;NQD2SR)+STRSV(1,1;NQD2SR)
      Y(ILOC1;NQD2SR)=Y(ILOC1;NQD2SR)+STRV(1,1;NQD2SR)
80     CONTINUE
       IF(ICON.EQ.1) GO TO 90
       IF(MP.EQ.0) GO TO 90
       IF(MC.EQ.1) GO TO 49
       WRITE(6,52)ITER
52     FORMAT(10X,'SOLUTION CONVERGED IN',I4,'ITERATIONS')
       ICON=1
       GO TO 49
53     WRITE(6,54)ITER
54     FORMAT(10X,'NO CONVERGENCE IN ',I4,'ITERATION')
       CALL PLOUT(NODE1,NODE2,NELE1,NELE2)
       GO TO 999
49     IF(ITER.EQ.MAXIT) GO TO 53
       IF(KLU.EQ.1.AND.NL.EQ.0) GO TO 90
50     CONTINUE
90     CONTINUE
       MP=0
       ICON=0
       IF(KLU.EQ.1) GO TO 36
91     CONTINUE
       CALL CONTACT
       IF(KNEW.EQ.1) GO TO 45
       IF(NPLOT.EQ.1) CALL PLOUT(NODE1,NODE2,NELE1,NELE2)
       IF(NPL.EQ.NEP) CALL PLOUT(NODE1,NODE2,NELE1,NELE2)
       IF(NPL.EQ.NEP) NPL=0
       IF(NTYP.EQ.1) CALL BREAK
       IF(NTYP.EQ.1) GO TO 100
       IF(KLU.EQ.1) GO TO 100
       IF(NPLOT.EQ.1.AND.WORD.EQ.GROW) CALL BREAK
100    CONTINUE
       IF(KLU.EQ.2) GO TO 999
       IF(KLU.EQ.3) GO TO 999
       IF(KLU.EQ.1) GO TO 35
       IF(NPLOT.EQ.1) GO TO 99
       GO TO 20
99     CONTINUE
102    DELP=-DELP
       READ(5,121)P,WORD
121    FORMAT(E10.3,1X,A4)
       IF(WORD.EQ.HALT) GO TO 999
       NPL=0
       MPLAS(1;MXGAUS)=0
       GO TO 20
999    RETURN
       END
         SUBROUTINE ERTA(A,B)
         DIMENSION A(6)
         X=A(1)
         Y=A(2)
         Z=A(3)
         XY=A(4)/2.
         YZ=A(5)/2.
         ZX=A(6)/2.
         S1=(X-Y)**2
         S2=(Y-Z)**2
```

```
      S3=(Z-X)**2
      S4=6*(XY)**2
      S5=6*(YZ)**2
      S6=6*(ZX)**2
      STOT=S1+S2+S3+S4+S5+S6
      SPO=SQRT(STOT)
      SO=SQRT(2.)/3.
      B=SO*SPO
      RETURN
      END
      SUBROUTINE PLOUT(NODE1,NODE2,NELE1,NELE2)
     COMMON/MAIN/AA(2000000),BB(9600,1),D(6,6),DINV(6,6),
    1DISP(80),EPS(12400),EPEST(1550),FORCE(10),LINE(100),
    2LOCAT(10),LBFOR(10),MB(9600),MSUM(9600),MPTAB(9600),
    3MPLAS(12400),NODE(8,1550),MPLC(1550),NODXO(700),
    4NODYO(700),NODZO(700),NODXC(700),NODYC(700),NODZC(700),
    5NODFIX(80),NODLOD(80),NDISP(80),R(9600),
    6SIGBAR(12400),SK(24,24),T1(9600),T2(9600),T4(2000),
    7T3(9600),U(3200),UOLD(3200),V(3200),VOLD(3200),V2(3200),
    8W(3200),WOLD(3200),X(74400),XR(3200),Y(74400),
    9YR(3200),Z(9600),ZR(3200)
     COMMON/CNST/EPSI,SK2,LMAX,KMAX,DAX,
    1PYLD,SCRIT,YOUNG,POIS,CRACK,PT,WIDTH,PMAX,HP,
    2SBAR,LPRIT,NGAUS,NLAYER,NNODE,
    3INODXO,INODYO,INODZO,INODXC,INODYC,INODZC,LNSTIF,MXNOD,
    4MXNEL,MXGAUS,ICUT,LTOTB,ITNODX,KLU,NTYP,NLM,
    5NDOF,KNEW,NEP,ERIT,NELM,AM,ROM

     COMMON/D382/NNPE,NDF,NQD,NSTR,NQD2,NNPE2,NQD2NPE,NQD2SR,MXQ2S
     COMMON/VECT/ STRV(8,6),STRSV(8,6),BMT(64,3),WDUM(8),XE(8,3),
    C NCUBE(8),DIS(8,3)
      DIMENSION STRS(6),FRC(24),FORCEX(3200),FORCEY(3200),FORCEZ(3200)
C
C *** OUTPUT ROUTINE
C
      WRITE(6,10)PT,CRACK,WIDTH
10    FORMAT(/,10X,'APPLIED LOAD=',E12.5,8X,'CRACK=',
    1 F10.5,10X,'WIDTH=',F10.5/)
      IF(CRACK.LT.EPSI) GO TO 20
      WRITE(6,15)
15    FORMAT(12X,'NODE',5X,'X',10X,'Y',10X,'Z',10X,
    1 'COD'/)
      CRACK1=CRACK+EPSI
      CRACK2=CRACK-10*DAX
      DO 16 I=1,INODYO
      L=NODYO(I)
      IF(XR(L).LT.CRACK2) GO TO 16
      IF(YR(L).GT.EPSI) GO TO 16
      IF(XR(L).GT.CRACK1) GO TO 16
      WRITE(6,25)L,XR(L),YR(L),ZR(L),VOLD(L)
25    FORMAT(5X,I4,4E14.6)
16    CONTINUE
20    CONTINUE
      WRITE(6,30)
30    FORMAT(//,30X,'DISPLACEMENTS'//12X,'NODE',14X,'U',
    1 18X,'V',18X,'W')
      DO 12 N=NODE1,NODE2
12   WRITE(6,22) N,UOLD(N),VOLD(N),WOLD(N)
22    FORMAT(10X,I5, 5X,3(E13.6, 3X))
      WRITE(6,35)
35    FORMAT(//20X,'STRESSES AND STRAINS',10X,1H*,3X,
    1 'DENOTES PLASTIC ELEMENTS',30X,'EFFECTIVE'//,5X,
    1 'ELEMENT',5X,'SIGX',8X,'SIGY',8X,'SIGZ',8X,'TAUXY',
    1 8X,'TAUYZ',8X,'TAUZX',8X,'EFFE-STRESS'//)
C
C *** CALCULATE NODAL FORCES AND GAUSS POINT STRESSES
```

```
C
      WRITE(6,25) NGAUS
      NGAUS=2
      FORCEX(1;NNODE)=0.0
      FORCEY(1;NNODE)=0.0
      FORCEZ(1;NNODE )=0.0
      N=0
      IGAUSP=0
      DO 300 IE=1,NELM
      NCUBE(1;8)=NODE(1,IE;8)
      CALL CORDIN(NCUBE,MXNOD,XR,YR   R,XE)
      ILOC1=(IE-1)*NQD2SR+1
      STRSV(1,1;NQD2SR)= X(ILOC1;N(   SR)
      CALL CDER(XE,NGAUS,BMT,WDUM,2
      CALL FORCEP(BMT,WDUM,STRSV,FR
      DO 352  I=1,8
      I1=NCUBE(I)
      IX=I*3-2
      FORCEX(I1)=FORCEX(I1)+FRC(IX)
      FORCEY(I1)=FORCEY(I1)+FRC(IX+
352   FORCEZ(I1)=FORCEZ(I1)+FRC(IX+
      DO 350 IG=1,NQD2
      IGAUSP=IGAUSP+1
      DO 351 I=1,6
351   STRS(I)=STRSV(IG,I)
      SIGO=(1-ERIT)*SIGBAR(IGAUSP)
       CALL SEQU(STRS,STP)
       IF(STP.GE.SIGO) GO TO 42
       WRITE(6,43)IE,IG,(STRS(L),L=   6),STP
43     FORMAT(5X,I6, I2,5X,7E12.5)
       GO TO 350
42     WRITE(6,45)IE,IG,(STRS(L),L=   6),STP
45     FORMAT(4X,1H*,I6, I2,5X,7E12   )
      KGAUSP=KGAUSP+1
350   CONTINUE
      IF(KGAUSP.LE.0) GOTO 300
93     N=N+1
       MPLC(N)=IE
300    CONTINUE
       NOPL=N
C
C ***
C
      WRITE(6,371)
      DO 370 IN=NODE1,NODE2
370   WRITE(6,372) IN,FORCEX(IN),F(   EY(IN),FORCEZ(IN)
371   FORMAT(1H1///10X,'NODE #',5X,   ORCEX',8X,'FORCEY',7X,'FORCEZ')
372   FORMAT(10X,I5,2X,3(E12.4,2X))
C
C ***
C
      WRITE(6,380)
380    FORMAT(//10X,'LIST OF PLASTI   ELEMENTS'//)
      WRITE(6,381) (MPLC(I),I=1,N)
381   FORMAT(5X,20I5)
997    RETURN
       END
       SUBROUTINE CONTACT
      COMMON/MAIN/AA(2000000),BB(96   ,1),D(6,6),DINV(6,6),
     1DISP(80),EPS(12400),EFEST(155   ,FORCE(10),LINE(100),
     2LOCAT(10),LBFOR(10),MB(9600),   UM(9600),MPTAB(9600),
     3MPLAS(12400),NODE(8,1550),MPL   1550),NODXO(700),
     4NODYO(700),NODZO(700),NODXC(7   ),NODYC(700),NODZC(700),
     5NODFIX(80),NODLOD(80),NDISP(8   ,R(9600),
     6SIGBAR(12400),SK(24,24),T1(9(   ),T2(9600),T4(2000),
     7T3(9600),U(3200),UOLD(3200),\   200),VOLD(3200),V2(3200),
```

```
      8W(3200),WOLD(3200),X(74400),XR(3200),Y(74400),
      9YR(3200),Z(9600),ZR(3200)
       COMMON/CNST/EPSI,SK2,LMAX,KMAX,DAX,
      1PYLD,SCRIT,YOUNG,POIS,CRACK,PT,WIDTH,PMAX,HP,
      2SBAR,LPRIT,NGAUS,NLAYER,NNODE,
      3INODXO,INODYO,INODZO,INODXC,INODYC,INODZC,LNSTIF,MXNOD,
      4MXNEL,MXGAUS,ICUT,LTOTB,ITNODX,KLU,NTYP,NLM,
      5NDOF,KNEW,NEP,ERIT,NELM,AM,ROM
C       CHANGES SPRING STIFNESS IF CRACK CLOSES OR OPENS.
C       DONT FORGET TO PUT THE COMMON HERE.
        WRITE(6,95)
95      FORMAT(5X,'CALLING FROM THE CONTACT')
        DO 10 I=1,INODYO
        L=NODYO(I)
        KNEW=0
        CX=XR(L)-CRACK
        IF(CX.LT.-EPSI) GO TO 9
        GO TO 40
9       NV=3*L-1
        MPTX=MPTAB(NV)
        IF(VOLD(L).LE.0.0) MPTAB(NV)=1
        IF(VOLD(L).GT.0.0) MPTAB(NV)=0
        IF(MPTX.NE.MPTAB(NV)) KNEW=1
        IF(KNEW.EQ.0) GO TO 40
        PN=PT-((PT-PMAX)/(VOLD(L)-V2(L)))*VOLD(L)
        Z(NV)=1.0
        NC=NV
        ALP=SK2
        IF(MPTAB(NV).EQ.0) ALP=-SK2
        IFAC=3
        CALL SYMBAN(LNSTIF,NDOF,MB,MSUM,AA,1,BB,IFAC,T1,IERR,
      1 ALP,Z,T2,T3,T4,NC)
        IF(MPTAB(NV).EQ.0) WRITE(6,20)L,PN
20      FORMAT(/,2X,'NODE',I3,'OPENED AT',F8.3)
        IF(MPTAB(NV).EQ.1) WRITE(6,30)L,PN
30      FORMAT(/,2X,'NODE',I3,'CLOSED AT',F8.3)
40      CONTINUE
10      CONTINUE
        RETURN
        END
        SUBROUTINE BREAK
       COMMON/MAIN/AA(2000000),BB(9600,1),D(6,6),DINV(6,6),
      1DISP(80),EPS(12400),EFEST(1550),FORCE(10),LINE(100),
      2LOCAT(10),LBFOR(10),MB(9600),MSUM(9600),MPTAB(9600),
      3MPLAS(12400),NODE(8,1550),MPLC(1550),NODXO(700),
      4NODYO(700),NODZO(700),NODXC(700),NODYC(700),NODZC(700),
      5NODFIX(80),NODLOD(80),NDISP(80),R(9600),
      6SIGBAR(12400),SK(24,24),T1(9600),T2(9600),T4(2000),
      7T3(9600),U(3200),UOLD(3200),V(3200),VOLD(3200),V2(3200),
      8W(3200),WOLD(3200),X(74400),XR(3200),Y(74400),
      9YR(3200),Z(9600),ZR(3200)
       COMMON/CNST/EPSI,SK2,LMAX,KMAX,DAX,
      1PYLD,SCRIT,YOUNG,POIS,CRACK,PT,WIDTH,PMAX,HP,
      2SBAR,LPRIT,NGAUS,NLAYER,NNODE,
      3INODXO,INODYO,INODZO,INODXC,INODYC,INODZC,LNSTIF,MXNOD,
      4MXNEL,MXGAUS,ICUT,LTOTB,ITNODX,KLU,NTYP,NLM,
      5NDOF,KNEW,NEP,ERIT,NELM,AM,ROM
C       COMMON GOES HERE.
        DO 8 I=1,INODYO
        L=NODYO(I)
        C2=ABS(ZR(L)-ZCOR)
        IF(C2.LT.EPSI) GO TO 10
        GO TO 8
10      CX=ABS(XR(L)-CRACK)
```

```
      IF(CX.LT.EPSI) GO TO 9
8     CONTINUE
9     IF(NTYP.EQ.0) GO TO 12
      JSUM=0
      DO 90 IO=1,INODYO
      NS=NODYO(IO)
      C1=ABS(ZR(NS)-ZCOR)
      IF(C1.LT.EPSI) GO TO 130
      GO TO 90
130   JSUM=JSUM+1
      LINE(JSUM)=NS
90    CONTINUE
      ITNODX=JSUM
      DIST=YOUNG
      DO 91 IP=1,ITNODX
      LM=LINE(IP)
      CSD=XR(L)-XR(LM)
      IF(CSD.LE.0.0) GO TO 91
      IF(CSD.GT.EPSI.AND.CSD.LT.DIST) DIST=CSD
91    CONTINUE
      ICAM=0
      MODF=LM
      DO 92 LU=1,INODYO
      C10=ABS(XR(LU)-XR(MODF))
      IF(C10.LT.EPSI) GO TO 96
      GO TO 92
96    ICAM=ICAM+1
92    LBFOR(ICAM)=LU
      LTOTB=ICAM
      DO 94 JO=1,LTOTB
      LA=LBFOR(JO)
      STR=VOLD(LA)
      WRITE(6,15)LA,STR,PT
15    FORMAT(2X,'CRACK-TIP NODE',I4,'HAD',E11.4,'CTOD AT'
     1 ,E11.4)
      KLU=0
      IF(STR.GE.(0.98*SCRIT))KLU=1
      IF(KLU.EQ.1) NBREAK=NBREAK+1
      IF(KLU.EQ.0) GO TO 997
94    CONTINUE
12    II=0
      DO 13 JJ=1,INODYO
      LL=NODYO(JJ)
      C3=XR(LL)
      C1=XR(L)
      C5=YR(L)
      C6=YR(LL)
      C7=ABS(C5-C6)
      C4=ABS(C3-C1)
      IF(C4.LT.EPSI.AND.C7.LT.EPSI) GO TO 155
      GO TO 13
155    II=II+1
      LOCAT(II)=LL
13    CONTINUE
      ICUT=II
      DO 16 JI=1,ICUT
      LC=LOCAT(JI)
      NV=3*LC-1
      MPTAB(NV)=0
      KLU=1
      IF(NBREAK.EQ.5) KLU=3
      ALP=-SK2
      Z(NV)=1.
      NC=NV
      IFAC=3
      CALL SYMBAN(LMSTIF,NDOF,MB,MSUM,AA,1,BB,IFAC,T1,IERR,
```

```
     1 ALP,Z,T2,T3,T4,NC)
       WRITE(6,100)LC,PT
100    FORMAT(/,2X,'NODE',I4,'BROKE AT',F8.3/)
16     CONTINUE
       CMIN=1.0E+10
       DO 36 JS=1,ICUT
       LO=LOCAT(JS)
36     FORCE(JS)=-SK2*VOLD(LO)
       DO 60 IJ=1,INODYO
       LT=NODYO(IJ)
       C5=XR(LT)-XR(L)
60     IF(C5.GT.EPSI.AND.C5.LT.CMIN) CMIN=C5
       CRACK=CRACK+CMIN
997    RETURN
       END
      SUBROUTINE SYMBAN(MAXN,N,M,MSUM,A,NRHS,B,IFAC,P,IERR,ALP,Z,W,D,T4, SYMBAN       2
     C NC)
C                                                                       SYMBAN       3
C    SOLVE MATRIX EQUATION AX=B WHERE A IS SYMMETRIC POSITIVE DEFINITE  SYMBAN       4
C    AND B IS A MATRIX OF CONSTANT VECTORS.                             SYMBAN       5
C    SYMMETRY AND BAND WIDTH OF MATRIX A IS ACCOUNTED FOR BY STORING A(I SYMBAN      6
C    IN LOWER TRIANGULAR MATRIX (INCLUDING DIAGONAL) BY ROWS            SYMBAN       7
C                                                                       SYMBAN       8
C    MAXN - MAXIMUM DIMENSION OF MATRIX A                               SYMBAN       9
C                                                                       SYMBAN      10
C    N - NUMBER OF ROWS OF MATRIX A                                     SYMBAN      11
C                                                                       SYMBAN      12
C    M(I) - BAND WIDTHS (LARGEST NUMBER OF NON-ZERO COLUMNS TO THE LEFT SYMBAN      13
C           AND INCLUDING THE DIAGONAL OF ROW I IN MATRIX A             SYMBAN      14
C           M(I) MUST BE GREATER THAN OR EQUAL TO 2 WITH M(1)=1         SYMBAN      15
C                                                                       SYMBAN      16
C    MSUM(I) - AN ARRAY COMPUTED IN SYMBAN                              SYMBAN      17
C                                                                       SYMBAN      18
C     T4(MXBND)  WORKING STORAGE OF LENGH MAX BAND WIDTH.
C
C     MXBND = MAX BAND WIDTH. T4 IS DIMENSIONED ONLY IN MAIN.
C
C    NRHS - NUMBER OF RIGHT-HAND SIDES OF COLUMN VECTOR B               SYMBAN      19
C                                                                       SYMBAN      20
C    IFAC - INPUT INTEGER SPECIFYING WHETHER OR NOT A CHOLESKY          SYMBAN      21
C          DECOMPOSITION OF MATRIX A IS TO BE COMPUTED                  SYMBAN      22
C          WHERE A = L*D*L**T                                           SYMBAN      23
C                                                                       SYMBAN      24
C          = 0    CHOLESKY DECOMPOSITION IS COMPUTED. IFAC SET TO 1.    SYMBAN      25
C                                                                       SYMBAN      26
C          = 1    THE CHOLESKY DECOMPOSED FORM OF MATRIX A IS INPUT.    SYMBAN      27
C                 SOLUTION IS RETURNED IN B.                            SYMBAN      28
C                                                                       SYMBAN      29
C          = 2    THE CHOLESKY DECOMPOSITION OF MATRIX A IS MODIFIED.   SYMBAN      30
C                 MODIFICATION IS OF THE FORM A = A + ALP*Z*Z**T.       SYMBAN      31
C                 NC IS THE FIRST NONZERO ROW IN COLUMN VECTOR Z.       SYMBAN      32
C                 IFAC IS SET TO 1 AND Z SET TO 0 UPON RETURN.          SYMBAN      33
C                 SOLUTION IS RETURNED IN B.                            SYMBAN      34
C                                                                       SYMBAN      35
C          = 3    ONLY THE CHOLESKY DECOMPOSITION OF MATRIX A IS MODIFIED SYMBAN    36
C                 IFAC IS SET TO 1 AND Z SET TO 0 UPON RETURN.          SYMBAN      37
C                                                                       SYMBAN      38
      DIMENSION A(MAXN),B(N,NRHS),P(N),W(N),D(N),Z(N),M(N),MSUM(N),T4(1) SYMBAN     39
      IF (IFAC.GT.0) GO TO 11                                           NEW        180
      LL=1                                                              SYMBAN      41
      DO 10 I=1,N                                                       SYMBAN      42
      MSUM(I)=LL                                                        NEW        181
      LL=LL+M(I)                                                        SYMBAN      44
   10 CONTINUE                                                          SYMBAN      45
   11 CONTINUE                                                          NEW        182
      IERR=0
```

```
      CALL QSOLV(A,MSUM,M,B,P,N,IFAC,D,W,Z,T4,ALP,NC,IERR)              NEW      183
      RETURN                                                           SYMBAN   48
      END                                                              SYMBAN   49
      SUBROUTINE QSOLV(AR,IB,IL,B,DI,N,NFACT,D,W,Z,T,ALP,NC,IERR)
      DIMENSION AR(1),IB(1),IL(1),B(1),DI(1),D(1),W(1),Z(1)             NEW
      DIMENSION T(1)                                                    NEW
      DESCRIPTOR AV,BV                                                 QSOLV
      IF(NFACT.GE.2) GO TO 300                                         NEW
      IF (NFACT.NE.0) GO TO 160                                        QSOLV
C     FACTOR                                                           QSOLV
      DO 100 I=1,N                                                     QSOLV
      ICI=I-IL(I)+1                                                    QSOLV
      T(1;IL(I))=AR(IB(I);IL(I))                                       QSOLV
      N1=IB(I)+I-ICI                                                   QSOLV
      AR(N1)=-1                                                        QSOLV
      DO 100 J=ICI,I                                                   QSOLV
      ICJ=J-IL(J)+1                                                    QSOLV
      KS=MAX0(ICI,ICJ)                                                 QSOLV
      N1=KS-ICI+1                                                      QSOLV
      N2=J-KS+1                                                        QSOLV
      ASSIGN AV,T(N1;N2)                                               QSOLV
      N1=IB(J)+KS-ICJ                                                  QSOLV
      ASSIGN BV,AR(N1;N2)                                              QSOLV
      C=Q8SDOT(AV,BV)                                                  QSOLV
      N1=J-ICI+1                                                       QSOLV
      T(N1)=-C                                                         QSOLV
      IF (J.EQ.I) GO TO 110                                            QSOLV
      N2=IB(I)+J-ICI                                                   QSOLV
      AR(N2)=T(N1)*DI(J)                                               QSOLV
      GO TO 100                                                        QSOLV
  110 CONTINUE                                                         QSOLV
      IF(T(N1).LE.0.0) GOTO 999
      DI(I)=1/T(N1)                                                    QSOLV
      D(I)=T(N1)
  100 CONTINUE                                                         QSOLV
C     FORWARD SUBSTITUTION                                             QSOLV
  160 CONTINUE                                                         QSOLV
      DO 200 I=1,N                                                     QSOLV
      ICI=I-IL(I)+1                                                    QSOLV
      ASSIGN AV,AR(IB(I);IL(I))                                        QSOLV
      ASSIGN BV,B(ICI;IL(I))                                           QSOLV
      C=Q8SDOT(AV,BV)                                                  QSOLV
      B(I)=-C                                                          QSOLV
  200 CONTINUE                                                         QSOLV
C     DIAGONAL                                                         QSOLV
      B(1;N)=B(1;N)*DI(1;N)                                            QSOLV
C     BACKWARD SUBSTITUTION                                            QSOLV
      NM1=N-1                                                          QSOLV
      DO 400 II=1,NM1                                                  QSOLV
      I=N-II+1                                                         QSOLV
      ICI=I-IL(I)+1                                                    QSOLV
      IF (ICI.GE.I) GO TO 400                                          QSOLV
      B(ICI;I-ICI)=B(ICI;I-ICI)-AR(IB(I);I-ICI)*B(I)                   QSOLV
  400 CONTINUE                                                         QSOLV
C     SOLUTION IS NOW IN B                                             QSOLV
C     D(1;N)=1.0/DI(1;N)                                               NEW
      RETURN                                                           QSOLV
C                                                                      NEW
C*********************  NFACT = 2 OR 3  ******************* ************ NEW
C                                                                      NEW
  300 CONTINUE                                                         NEW
      DO 310 J=NC,N                                                    NEW
      D(J)=D(J)+ALP*Z(J)*Z(J)                                          NEW
      BETA=ALP*Z(J)/D(J)                                               NEW
      ALP=ALP/(D(J)*DI(J))                                             NEW
      DI(J)=1.0/D(J)                                                   NEW
```

```
      IF(J.EQ.N) GO TO 310                                                    NEW
      JN=J+1                                                                  NEW
      DO 311 I=JN,N                                                           NEW
      IF(I-J.GT.IL(I)-1) GO TO 311                                            NEW
      K=IB(I)+IL(I)-1+J-I                                                     NEW
      Z(I)=Z(I)-Z(J)*AR(K)                                                    NEW
      AR(K)=AR(K)+BETA*Z(I)                                                   NEW
  311 CONTINUE                                                                NEW
  310 CONTINUE                                                                NEW
      Z(1;N)=0.0                                                              NEW
      IF(NFACT.EQ.3) GO TO 320                                                NEW
      NFACT=1                                                                 NEW
      GO TO 160                                                               NEW
  320 CONTINUE                                                                NEW
      NFACT=1                                                                 NEW
      RETURN                                                                  NEW
999   IERR=1
      RETURN
      END                                                                     QSOLV
      SUBROUTINE ZEROLV(A,L)
      DIMENSION A(L)
      DATA LPAGE/65535/
      IF(L.LE.LPAGE) GO TO 10
      N=L/LPAGE
      LEFT=L-(L/LPAGE)*LPAGE
      DO 20 I=1,N
      LFIRST=LPAGE*(I-1)+1
      A(LFIRST;LPAGE)=0.0
   20 CONTINUE
      LFIRST=LPAGE*N+1
      A(LFIRST;LEFT)=0.0
      RETURN
   10  A(1;L)=0.0
      RETURN
      END
```

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